You have 150 minutes.

No books, calculators, computers, etc. are allowed. However, a "crib sheet" is allowed, subject to the rules stated earlier: one side of one standard sheet of paper, written/typed/drawn by you, etc.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Do not write "+" to mean "and". Define any notation that you introduce. For example, if you write "$P(A)$", but neither I nor you have defined an event A, then that writing is not good.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic unless it is specifically requested. Mark your final answer clearly.

Good luck.

A. Recall that a Rayleigh-distributed Y has PDF $f_Y(y) = ye^{-y^2/2}$ on support $(0, \infty)$. Suppose that you have a way of generating random samples from any continuous uniform distribution $\mathrm{Unif}(a, b)$. Describe in detail how you can then generate random values of Y.

B. Let X and Y be independent continuous random variables. Assume that X and Y are always positive. Let $T = Y^X$. Mimicking the development of convolution, find an expression for the PDF of T in terms of the PDFs of X and Y.

C.A. What is the MGF of $X \sim \text{Geom}(p)$? (You must derive the answer from the definition of MGF. You may not simply state the answer from your memory or crib sheet.)

C.B. What is the MGF of $Y \sim \text{NBinom}(r, p)$? (Again, you may not simply state the answer.)

C.C. According to the central limit theorem, a certain transformation Z of Y has $m_Z(t) \to e^{t^2/2}$ as $r \to \infty$. What is Z in terms of Y?

D. Let X and Y be two random variables and $h(x)$ any function.

D.A. Show that $E(Y|X) \cdot h(X) = E(Y \cdot h(X)|X)$.

D.B. Show that $Y - E(Y|X)$ is uncorrelated with $h(X)$. (This fact is important to how conditional expectation is used in regression.)

E. You run a web site that receives 1,000 visitors per day on average. Working within this context, for each of the following distributions, ask a question whose answer is a random variable from that distribution, exactly or approximately. Be sure to specify how the parameters of the distribution depend on the information given and any other necessary information.

E.A. Poisson:

E.B. Exponential:

E.C. Normal:

E.D. Uniform:

E.E. Bernoulli: