This exam is intended to take no more than 70 minutes. I am allowing 150 minutes to account for typing/recopying of solutions, technical problems, etc.

The exam is open-book and open-note — meaning:

- You may use all of this course's materials: the Tapp textbook, your class notes, your old homework, and the course web/Moodle site. You may also view official Mathematica reference/tutorial material, as long as it has nothing to do with differential geometry.
- You may not share any materials with any classmates while you are taking the exam. You may not consult any other books, papers, Internet sites, etc. You may not discuss the exam in any way — spoken, written, etc. — with anyone but me, until everyone has handed in the exam.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. Always show enough work and justification so that a typical classmate could understand your solutions. Feel free to cite material (definitions, theorems, examples, etc.) from class, the assigned textbooks readings, and the assigned homework problems. On the other hand, you may not cite results that we have not studied.

If you cannot solve a problem, then write a brief summary of the approaches you've tried. Partial credit is often awarded. Correct answers without supporting work rarely earn full credit.

I will try to check my e-mail frequently during the exam period. Feel free to ask clarifying questions. If you cannot obtain clarification on a problem, then explain your interpretation of the problem in your solution. Never interpret a problem in a way that renders it trivial. Check your e-mail occasionally, in case I send out a correction.

Good luck. :)

For the first problem, let $\vec{\gamma}, \vec{\delta} : \mathbb{R} \to \mathbb{R}^n$ be unit-speed curves that stay a constant distance away from each other — that is, there is a positive constant c such that

$$\left|\vec{\gamma}(t) - \vec{\delta}(t)\right| = c$$

for all t.

A. Prove that the angle between $\vec{\gamma}'(t)$ and the difference vector $\vec{\gamma}(t) - \vec{\delta}(t)$ is always equal to the angle between $\vec{\delta}'(t)$ and the difference vector.

For the next problem, let $\vec{\gamma} : [-2\pi, 2\pi] \to \mathbb{R}^2$ be a simple, closed, unit-speed curve. Define curves $\vec{\delta}, \vec{\beta}, \vec{\epsilon} : [-2\pi, 2\pi] \to \mathbb{R}^2$ by

$$\vec{\delta}(t) = \vec{\gamma}'(t), \quad \vec{\beta}(t) = \int_0^t \vec{\gamma}(u)\, du, \quad \vec{\epsilon}(t) = (\cos t, \sin t).$$

B.A. Is $\vec{\delta}$ necessarily simple? Closed? Unit-speed?

B.B. Is $\vec{\beta}$ necessarily simple? Closed? Unit-speed?

B.C. Prove that $\left|\vec{\delta}(t) + \vec{\epsilon}(t)\right| = 0$ at some time $t \in [-2\pi, 2\pi]$.

For the next problem, consider a curve $\vec{\gamma} : \mathbb{R} \to \mathbb{R}^3$ that is parametrized by arc length, with $\kappa > 0$ constant and τ constant.

C.A. Prove that $\vec{t}''' = -c^2 \vec{t}'$ for some positive constant c. Also, give an explicit expression for c.

C.B. Find an explicit expression for $\vec{\gamma}$. It will involve some undetermined constants. [Hint: In calculus, a function $y = f(x)$ satisfies $y'' = -c^2 y$ if and only if y is a linear combination of $\sin(cx)$ and $\cos(cx)$.]