A. We cannot make a perfectly accurate map of the space station's surface, for the same reason that we cannot make a perfectly accurate map of the Earth.

In detail, we have seen a torus where the inner radius is 1 and the outer radius is 3 (so that circles of radius 1 have been revolved to form the torus as a surface of revolution). The Gaussian curvature varies from -1 on the inner rim to 0 on the top and bottom to $1/3$ on the outer rim. Because the Gaussian curvature is not 0, there does not exist an isometry between the torus and the plane. Now, the proportions of the space station might not match this example, but we could presumably generalize the example and reach the same conclusion about the space station.

Because there does not exist an isometry, there does not exist a map that is both equiareal and conformal. In any map, either areas or angles will be misrepresented (or both). If Babatope cares primarily about areas (for example, helping the government allocate space for competing entertainment venues), then they should try to construct an equiareal map. If they care primarily about angles (for example, helping the government plan how links in a transportation network meet), then they should try to construct a conformal map. Otherwise, perhaps they should try to construct a map that compromises between the two desiderata.

[You are not expected to devise an equiareal or conformal parametrization to solve Babatope's problem. But for the sake of your education you might think about how you would try one of those.]

B.A. Yes. The first fundamental form is the function that takes $\vec{v} \in \mathbb{R}^2$ to $\vec{v}^\top \cdot \mathbf{G} \cdot \vec{v} \in \mathbb{R}$.

B.B. No. If the second fundamental form were intrinsic, then its extrema would be intrinsic. But those are the principal curvatures k_1 and k_2, which we have seen not to be intrinsic by comparing a plane with a cylinder.

C.A. [There was an error in my typed solutions. I have not fixed it yet.]

C.B. There are six independent Christoffel symbols, so we expect to need six linear equations. (We might need more, if some of the equations ended up being redundant.) To get four more equations, we might try expanding

$$\frac{\partial}{\partial x_2}\frac{\partial \tilde{\vec{f}}}{\partial x_1}, \qquad \frac{\partial}{\partial x_2}\frac{\partial \tilde{\vec{f}}}{\partial x_2}$$

each in two ways.

C.C. It is not true that $\mathbf{G}(\vec{p}) = \tilde{\mathbf{G}}(\vec{p})$. The simplest reason is that $\vec{f}$ and $\tilde{\vec{f}}$ might map $\vec{p}$ to wildly different parts of the surface. So what if we correct for that, by asking whether $\mathbf{G}(\vec{p}) = \tilde{\mathbf{G}}(\vec{\phi}(\vec{p}))$? [Actually, that's what I meant to ask.] That's not good enough either. Some correction has to be made for the differential of $\vec{\phi}$, using the chain rule. More concretely, we have seen multiple parametrizations of $\mathbb{S}^2$ with differing expressions for $\mathbf{G}$. In general, $\mathbf{G}$ depends not just on how the surface S sits in $\mathbb{R}^m$ but also on how $\vec{f}$ parametrizes that S.