The first problem proves a fact that we've used in lecture. The second problem has three answers so far, and we'll see another answer soon. The other two problems are used in lectures in the near future.

A. Prove, for any real number t, that $\left|e^{i2t} - 1\right|^2 = 4\sin^2 t$. (You might need to use the double-angle identity $\cos(2t) = 1 - 2\sin^2 t$ from trigonometry.)

B. Why does Shor's algorithm insist on using an n large enough that $2^n \geq m^2$?

C. Let c/d and c'/d' be distinct rational numbers in lowest terms (meaning $\gcd(c, d) = 1 = \gcd(c', d')$). Assume that $d, d' < m$ and $2^n \geq m^2$. Prove that the distance between these two numbers (on the real number line) is greater than 2^{-n}.

D. Suppose that a and b are distinct large primes. You don't know them, but you know $m = ab$ and $\varphi(m)$. Explain how to deduce a and b from m and $\varphi(m)$.