This problem is about the three-qbit error-correcting protocol for X errors. One quarter of the problem was done in lecture.

A. For each of the four possible error circuits, work out the effect of F (detection) and the effect of G (correction). In particular, it should be clear how the syndrome (the value of the fourth and fifth qbits) signals which error circuit happened.

The following exercise establishes the concept of a *controlled-U* gate CU. We could have done this exercise much earlier in the course, but we didn't, because we didn't need it until error correction.

B. Let U be any n-qbit gate. Define a function $CU : \mathbb{C}^{2^{n+1}} \to \mathbb{C}^{2^{n+1}}$ by declaring, for any n-qbit state $|\psi\rangle$, that

$$CU(|0\rangle \otimes |\psi\rangle) = |0\rangle \otimes |\psi\rangle \text{ and } CU(|1\rangle \otimes |\psi\rangle) = |1\rangle \otimes U \cdot |\psi\rangle\,.$$

Find the $2^{n+1} \times 2^{n+1}$ matrix that represents CU, and prove that that matrix is unitary.