

This homework will not be collected. But you are expected to do it. Consider it a couple of study questions for Exam C. The questions are intentionally open-ended.

A. For the Steane (1996) seven-qubit code, how can we understand that the encoding circuit functions as advertised? How much work would it take to verify this claim in Python? Alternatively, how much work would it take to verify this claim algebraically? The H gates on the last three wires generate a state of the form $2^{-3/2}(|000\rangle + \dots + |111\rangle)$. For one or two terms in that superposition, analyze what the rest of the circuit does. If you did this for all eight terms, then would your verification be complete?

B. For the $-R$ gate in Grover's algorithm, I claimed that, other than layers of H and X gates, all that is needed is an n -qubit multiply-controlled- Z gate. I verified this claim for the $n = 2$ case. Now, verify it for the $n = 3$ case. Also, think about how you would prove it in general. It might help to consider this question: For arbitrary one-qubit U , is it true that

$$cc \cdots cU = c(c \cdots cU)?$$

(On the left side of that equation is an n -qubit gate, in which the first $n - 1$ qubits jointly control a U gate on the last qubit. On the right side of the equation is an n -qubit gate, in which the first qubit controls a multiply-controlled- U gate on the last $n - 1$ qubits.)