There are six pages including this cover page. You have 70 minutes. No notes, books, calculators, computers, etc. are allowed.

Show your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit. Perform as much algebraic simplification as you can.

You may cite without proof any result discussed in class, the assigned textbook sections, or the assigned homework.

Good luck. :)

A. Here are some miscellaneous, concrete calculations. Remember to simplify.

A.A. $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ i\frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{\sqrt{3}}{2} - \frac{1}{2}i \\ 0 \end{bmatrix} =$

A.B. $X \otimes H =$

B. In class we have seen the construction $f \cdot |\alpha\rangle\,|\beta\rangle = |\alpha\rangle\,|\beta \oplus f(\alpha)\rangle$.

B.A. What type of thing is α?

B.B. What type of thing is $|\alpha\rangle$?

B.C. What type of thing is f, in its first (left-most) occurrence?

B.D. What type of thing is f, in its second (right-most) occurrence?

C. This question is about the states $|\alpha\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$ and $|\beta\rangle = \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$.

C.A. When $|\alpha\rangle$ is measured, what are the possible results, and with what probabilities? Then answer the same question for $|\beta\rangle$.

C.B. Does there exist a one-qbit quantum algorithm that behaves differently on those two inputs? Prove not or give an example.

D. This question is about the two-qbit circuit SWAP $\cdot$ CNOT $\cdot$ SWAP.

D.A. Draw a circuit diagram.

D.B. Compute the matrix for the circuit. Have we ever seen this matrix in our course thus far?

E. These are TRUE-FALSE questions. No explanation is needed. A correct answer earns 2 points, and an incorrect answer earns 0 points. You may optionally answer PUNT, which earns 1 point. Write clearly; an illegible or ambiguous answer earns 0 points.

E.A. Any magnitude-1 complex number is of the form $e^{i\theta}$ for some real θ.

E.B. Any unitary $N \times N$ matrix is invertible.

E.C. Any classical two-qbit state is unentangled.

E.D. Any classical one-bit gate can be implemented as a quantum one-qbit gate.

E.E. Any classical two-bit gate can be implemented as a quantum two-qbit gate.

E.F. Any two-qbit state is a linear combination of unentangled two-qbit states.

E.G. Partial measurement of any two-qbit state produces a classical two-qbit state.

E.H. Partial measurement of any two-qbit state produces an unentangled two-qbit state.

F. This question is about Deutsch's algorithm for solving Deutsch's problem.

F.A. What is Deutsch's problem?

F.B. What is Deutsch's algorithm? You need not explain why it works. A complete answer is not necessarily long and preferably short.