Recall that

$$CLIQUE = \{\langle G, k\rangle : G \text{ is an undirected graph, } k \geq 1, \text{ and } G \text{ contains a } k\text{-clique}\}.$$

Also, for any $k \geq 1$, let

$$CLIQUE_k = \{\langle G\rangle : G \text{ is an undirected graph that contains a } k\text{-clique}\}.$$

In class, we will soon learn that $CLIQUE$ is NP-complete. Without going into details, this fact implies that if $CLIQUE \in P$, then $P = NP$. The popular belief is that $P \neq NP$ and hence $CLIQUE \notin P$.

A. Show that $CLIQUE_k \in P$ for all k. (For the sake of Problem B, it might help if you try to pin down your running time fairly precisely. By the way, the $k = 3$ case is Problem 7.9 in our textbook.)

B. Explain how it's possible that $CLIQUE_k \in P$ for all k, but $CLIQUE \notin P$. In other words, explain why someone might think that $(\forall k\ CLIQUE_k \in P) \Rightarrow CLIQUE \in P$, and why that argument can't be completed.