This exam consists of Problems A-G, spread over six pages (not including this front page or the blank back page).

Notes, book, etc. are not allowed.

If you feel that a problem is ambiguously worded, then ask for clarification. If the problem is still unclear, then explain your interpretation in your solution. Never interpret a problem in a way that renders it trivial.

Except where otherwise noted, you should always justify your answers. Correct answers with no justification may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit.

It is understood that efficient, concise solutions are usually favored over inefficient or verbose solutions, and hence may earn more points.

You have 150 minutes. Good luck.

A. Suppose that a language A is decidable by a nondeterministic Turing machine N in space $s(n)$. Estimate the time required to decide A on a deterministic Turing machine.

B. I want to prove that Python is not regular. So I assume for the sake of contradiction that it is regular, and let p be a pumping length. Tell me a string that I should pump, to derive my contradiction. You do not need to complete the proof or justify your answer.

C. Suppose, for the sake of argument, that you have a decider H for $\mathrm{HALT}_{\mathrm{TM}}$. Give an algorithm that uses H to compute the Kolmogorov complexity $K(x)$ of any string x.

D. This three-part problem concerns the following nine computability and complexity classes. No justification is needed on this problem.

the context-free languages	the finite languages	the regular languages
NP	the recognizable languages	the decidable languages
EXPTIME	P	PSPACE

D1. Put those nine classes into order, from the smallest class to the largest class, using "$\subseteq$".

D2. Give a language that is not in the decidable languages but is in the next larger class.

D3. We suspect, but do not know, that NP is truly bigger than the next smaller class. Give a language that seems to be in NP but not in the next smaller class.

E. Suppose that $A \leq_p B$ and B is context-free. Give the smallest class that we have studied, that is guaranteed to contain A. (Examples of "classes" are listed in Problem D.)

F. In each part of this problem, there are four valid answers: TRUE, FALSE, WEAK TRUE, and WEAK FALSE. The correct answer earns full credit. The weak correct answer earns 2/3 credit, the weak incorrect answer earns 1/3 credit, and the incorrect answer earns no credit. Justification is not required on this problem.

1. If a Turing machine M has exponential time complexity, then M halts on all inputs.

2. If a two-tape Turing machine M has polynomial time complexity, then there exists an equivalent one-tape Turing machine of polynomial time complexity.

3. If $A \leq_p B$ and A is NP-complete, then B is NP-complete.

4. Our proof of the Cook-Levin theorem uses a divide-and-conquer strategy.

5. If P = PSPACE, then every $A \in$ P is PSPACE-complete.

6. If A and $\bar{A}$ are both recognizable, then both of them are decidable too.

G1. You're eating lunch with President Poskanzer. He asks you, "What is TQBF?" (A nontrivial example would probably help explain.)

G2. Then he asks you, "Why is TQBF important to computer science?"