

A.A. For each string $w \in A$, it is easy to construct a DFA that accepts w and no other strings. Then we can construct an NFA for A by creating a start state and adding an ϵ -transition from that start state to the start state of each DFA.

A.B. Because A is finite, there is a longest string x (breaking ties arbitrarily). I claim that the minimal pumping length is $p = |x| + 1$. The pumping lemma is trivially true for this p , because there are no strings $w \in A$ such that $w \geq p$. The pumping lemma is not true for $p = |x|$, because then the string x could be pumped to produce infinitely many strings in A .

B. My regular expression is

$$(\epsilon \cup -)DD^*(\epsilon \cup .DD^*)(\epsilon \cup (e \cup E)(\epsilon \cup -)DD^*).$$

[The first chunk matches an optional $-$. The second chunk matches one or more digits — the integral part of the number. The next chunk matches an optional fractional part of the number. The final chunk matches an optional e or E construction.]

C.A. TRUE. [Let G be a CFG for A with start variable S . Construct a new CFG with start variable S' , rule $S' \rightarrow \epsilon|SS'$, and all the rules of G .]

C.B. FALSE. [We drew only three states in our construction, but there may be many additional states, hanging off the middle state, needed to push strings of variables and terminals.]

C.C. FALSE. [Our PDAs are non-deterministic. The book also discusses DPDAs, which we mentioned in class only briefly.]

C.D. TRUE. [This was a homework problem. Use structural induction on the definition of regular expression.]

C.E. FALSE. [The “ $|uvx| \leq p$ ” part should read “ $|vxy| \leq p$ ”.]

D. Assume for the sake of contradiction that A is regular. Let p be a pumping length for A . Let $w = 0^p10^{2p}$. Then $w \in A$ and $|w| \geq p$, so the pumping lemma guarantees the existence of strings x, y, z satisfying certain properties. Because $|xy| \leq p$, it must be true that $xy = 0^k$ for some $k \leq p$. Because $|y| \leq 1$, it must be true that $y = 0^\ell$ for some ℓ such that $1 \leq \ell \leq p$. Then $xy^2z = 0^{p+\ell}10^{2p} \notin A$. This contradiction implies that our original assumption was false. Therefore A is not regular.

E. [I won't draw the PDA here, but I will explain the key ideas.] First draw a PDA P_1 for $A_1 = \{a^i b^j c^k : i \neq j\}$. Then draw a PDA P_2 for $A_2 = \{a^i b^j c^k : j \neq k\}$. Then add a new start state and ϵ -transitions from the new start state to the start states of P_1 and P_2 . The resulting PDA accepts $A_1 \cup A_2 = A$.