A. [We discussed this issue in class, right after defining $PSPACE$-hardness.] Notice that $P \subseteq NP$ and $P \subseteq PSPACE$. We always define our reductions to come from a smaller complexity class. In defining NP-hardness, we used reductions in P. In defining $PSPACE$-hardness, we again use reductions in P.

If we don't require our reductions to come from a smaller complexity class, then we produce trivial and uninteresting concepts. For example, say that a language B is "$PSPACE$-firm" if every language $A \in PSPACE$ can be polynomial-space mapping reduced to B. Then every language in $PSPACE$ is $PSPACE$-firm. [Why?] So it's not a definition worth pursuing.

B. [We discussed this non-proof in class, just before doing the proof.] How big would the tableau have to be? Its width would be $s(n)$, which is fine. Its height would be $t(n)$, meaning the time used by the NTM on inputs of size n. The problem is that $t(n)$ can be exponential in $s(n)$ (because of our bound on the number of configurations). So the size of the tableau and the resulting formula might be exponential in $s(n)$.

C. Assume for the sake of contradiction that A is decidable. Let D be a decider for A. We will describe a decider E for $EMPTY_{TM}$. On input $\langle M \rangle$, E does these steps:

1. Build a Turing machine N whose start state is its reject state. That is, N rejects all inputs immediately.

2. Run D on $\langle M, N \rangle$ and output the same result as D does.

This E is a decider because its first step is extremely simple and its second step is to run a decider. Notice that E accepts $\langle M \rangle$ if and only if $L(M) = L(N)$. But $L(N) = \emptyset$, so E accepts $\langle M \rangle$ if and only if $L(M) = \emptyset$. Thus E decides $EMPTY_{TM}$. But $EMPTY_{TM}$ is undecidable by Rice's theorem. From this contradiction we conclude that A cannot be decidable either.

D. [I won't draw the Venn diagram, but I will write out the crucial containments symbolically.]
REGULAR $\subseteq$ CONTEXTFREE, because every regular language is context-free.
CONTEXTFREE $\subseteq$ P, as we proved in class using dynamic programming.
P $\subseteq$ NP, because every TM is trivially an NTM.
NP $\subseteq$ NPSPACE, because time used always upper-bounds space used.
NPSPACE $=$ PSPACE as a consequence of Savitch's theorem.
PSPACE $\subseteq$ EXPTIME, as we proved in class by bounding the number of configurations.
EXPTIME $\subseteq$ EXPSPACE, because time used always upper-bounds space used.
EXPSPACE $\subseteq$ DECIDABLE; any language decidable in exponential space is decidable.

DECIDABLE is the intersection of RECOGNIZABLE and CORECOGNIZABLE, so it is a subset of both of those. There is no containment relationship between RECOGNIZABLE and CORECOGNIZABLE.

E. Let A be a regular expression matching all single characters other than carriage returns. Let R be a regular expression matching the carriage return character. Let D be a regular expression matching all digits. Let C be a regular expression matching all single characters other than carriage returns and commas. Let L be a regular expression matching all single letter characters.

The first line is matched by A^*.

The second line is matched by $(DD^*A^*) \cup (\texttt{PO Box}DD^*)$.

The third line is matched by $C^*\texttt{,}LLDDDDD(\epsilon \cup \texttt{-}DDDD)$.

Therefore the regular expression for matching whole addresses is

$$A^*R(DD^*A^*) \cup (\texttt{PO Box}DD^*)RC^*\texttt{,}LLDDDDD(\epsilon \cup \texttt{-}DDDD).$$

F. I will prove that A is not context-free. Then it follows that A is not regular either.

Assume for the sake of contradiction that A is context-free. Let p be a pumping length for A. Let $s = a^p b^p c^p$. Then s decomposes into $s = uvxyz$ in the usual way. Because $|vxy| \leq p$, we know that vxy is a substring of either $a^p b^p$ or $b^p c^p$.

If vxy is a substring of $a^p b^p$, then $uv^0xy^0z = uxz$ is of the form $a^k b^\ell c^p$. Because vy is nonempty, we know that $k < p$ or $\ell < p$ (or both). Thus $uv^0xy^0z \notin A$.

Similarly, if vxy is a substring of $b^p c^p$, then uv^0xy^0z is of the form $a^p b^k c^\ell$, where $k < p$ or $\ell < p$. Thus $uv^0xy^0z \notin A$.

In both cases we have pumped the string w out of A. This result contradicts the pumping lemma. We conclude that A cannot be context-free.