Notes, book, etc. are not allowed.

Except where otherwise noted, you must justify your answers. Correct answers with no justification may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit. For example, if you know what form an argument should take, but you can't fill in the details, then at least show me the form.

You may cite material (definitions, theorems, algorithms, etc.) discussed in class, the assigned homework, or the assigned textbook sections. If you wish to use other material, then you must develop it first.

It is understood that efficient, concise solutions are usually favored over inefficient or verbose solutions, and hence may earn more points.

If you feel that a problem is ambiguously worded, then ask for clarification. If the problem is still unclear, then explain your interpretation in your solution. Never interpret a problem in a way that renders it trivial.

You have 70 minutes. Good luck. :)

In class we used the undecidability of ACC_{TM} to prove the undecidability of $HALT_{TM}$. But we could have developed these results in the opposite order.

A. Suppose that we don't yet know that ACC_{TM} is undecidable. Prove that ACC_{TM} is undecidable using the fact that $HALT_{TM}$ is undecidable.

B.A. Our basic space-time lemma says that, for any deterministic Turing machine M, $s(n)$ is $\mathcal{O}(t(n))$. Does the conclusion hold for nondeterministic Turing machines also?

B.B. Suppose that a language A is decidable on a nondeterministic Turing machine N in time $t_N(n) \geq n$. Give an upper bound, in terms of $t_N(n)$, on how much space $s_D(n)$ is required to decide A on a deterministic Turing machine D.

C. In each part of this problem, M denotes a (deterministic, one-tape) Turing machine. Your job is to determine whether the given language is decidable, recognizable, and/or co-recognizable. Draw a box around the adjective if it applies to the language, and X-out the adjective if it does not apply. No explanation is necessary. The first part has been done for you. (Each adjective will be graded separately from the others. Missing and ambiguous markings will be graded as wrong. For a few adjectives, I don't know the answer myself; those adjectives will be removed from the grading altogether.)

C.A. $\{\langle M, w\rangle : M \text{ accepts input } w\}$

~~decidable~~

[recognizable]

~~co-recognizable~~

C.B. $\{\langle M, w\rangle : M \text{ never moves left on input } w\}$

decidable

recognizable

co-recognizable

C.C. $\{\langle M\rangle : M \text{ halts on input } \epsilon\}$

decidable

recognizable

co-recognizable

C.D. $\{\langle M, c\rangle : c \text{ is a positive integer, and for all inputs } M \text{ halts in at most } c \text{ steps}\}$

decidable

recognizable

co-recognizable

C.E. $\{\langle M \rangle : L(M)$ is decidable$\}$
decidable
recognizable
co-recognizable

C.F. $\{\langle M \rangle : \exists$ TM N such that the state diagram of N is not connected and $L(N) = L(M)\}$
decidable
recognizable
co-recognizable

C.G. $\{\langle M \rangle : M$ accepts $\langle M \rangle\}$
decidable
recognizable
co-recognizable

C.H. $\{\langle M, x \rangle : x$ is a valid input for $M\}$
decidable
recognizable
co-recognizable