A.A. [Instead of drawing, I'll describe in words. P $\hookrightarrow$ NP because every TM is an NTM. PSPACE $\hookrightarrow$ NPSPACE for the same reason. P $\hookrightarrow$ PSPACE by the space-time lemma. NP $\hookrightarrow$ NPSPACE and EXPTIME $\hookrightarrow$ EXPSPACE for the same reason. NP $\hookrightarrow$ EXPTIME by our usual NTM-on-TM simulation. NP $\hookrightarrow$ PSPACE for the same reason. NPSPACE $\hookrightarrow$ PSPACE by Savitch's theorem. PSPACE $\hookrightarrow$ EXPTIME by counting configurations.]

A.B. [Instead of drawing, I'll describe in words: RegLs $\subseteq$ CFLs $\subseteq$ P $\subseteq$ EXPSPACE $\subseteq$ DecLs. And DecLs is the intersection of RecLs and CoRecLs.]

B. [Instead of drawing, I'll describe in words. There are three states. There is a transition from the first state to the second state labeled "$\llcorner\!\lrcorner \mapsto \#, \mathcal{R}$". There is a transition from the second state to the third state with the same label. There is a transition from the second state to itself with two labels: "$\llcorner\!\lrcorner \mapsto 0, \mathcal{R}$" and "$\llcorner\!\lrcorner \mapsto 1, \mathcal{R}$".

C. Suppose that $A \leq_P B$ and $B \in NP$. So there exists a polynomial-time TM F such that $w \in A \Leftrightarrow F(w) \in B$, and there exists a polynomial-time NTM N that decides B. Here is an NTM M to decide A. On input w:

1. Run F on w to obtain the string $F(w)$.
2. Run N on $F(w)$ and output whatever N outputs.

This NTM M decides A, because

$$w \in A \Leftrightarrow F(w) \in B \Leftrightarrow N \text{ accepts } F(w) \Leftrightarrow M \text{ accepts } w.$$

The time taken by step 1 is bounded by n^k for some k (and for large n). The length of $F(w)$ is bounded by n^k, by the basic space-time lemma. The time taken by the step 2 is bounded by $(n^k)^\ell$ for some ℓ. So M is polynomial-time. Thus $A \in NP$.

D. First I fill the diagonal (where $i = j$), then the diagonal above that, then the diagonal above that, then the diagonal above that, and finally the top right cell. I use $\emptyset$ to denote that no variables of G can generate the desired substring. In the end, the lack of S in the top right cell means that S cannot generate w, and hence that $w \notin L(G)$. That is, the polynomial-time algorithm rejects w.

	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$i=1$	A, B	A, S	S	$\emptyset$	$\emptyset$
$i=2$		A, B	S	$\emptyset$	$\emptyset$
$i=3$			B	$\emptyset$	$\emptyset$
$i=4$				A, B	S
$i=5$					B

E.A.

q_0	0	1
1	q_1	1

0	q_2	1
q_{rej}	0	1

E.B.

0	q_2	0
q_1	0	0

F. If the NTM N uses space $s_N(n)$, then Savitch's theorem says that there is an equivalent TM M that uses space $\mathcal{O}(s_N(n)^2)$. As we have discussed in class, the number of possible configurations of M is then $2^{\mathcal{O}(s_N(n)^2)}$, and M cannot reuse a configuration without entering an infinite loop, so the time complexity of M is

$$t_M(n) = 2^{\mathcal{O}(s_N(n)^2)}.$$