Here's one thing that we didn't discuss in class today, that is relevant to the last few problems below: It is common to talk about points using the same notation as vectors. For example, in $\mathbb{R}^2$ consider the vector $\vec{v} = \langle 3, -2 \rangle$. If you place this vector at the origin, then it points to the point $(3, -2)$. When we talk about the *point* $\vec{v}$, we mean the point $(3, -2)$. More generally, the point $\vec{p} = \langle p_1, p_2 \rangle$ means the point (p_1, p_2).

Also, when you are asked to prove something, *prove* means "give an explanation that would convince a skeptical classmate".

1. Section 12.2 #19.

2. Section 12.2 #23.

3. Section 12.2 #30.

4. Section 12.2 #45. [For some students, this problem foreshadows the first week of Math 232. But you don't need to have studied linear algebra, to solve it.]

5. Let $\vec{p}$ and $\vec{q}$ be any two points in $\mathbb{R}^3$. Let $\vec{m} = \frac{1}{2}(\vec{p} + \vec{q})$. Using basic properties of vector addition and scalar multiplication, prove that $\vec{m}$ is the midpoint of the line segment between $\vec{p}$ and $\vec{q}$. [Hint: Compare the vector $\vec{p} - \vec{m}$ to the vector $\vec{q} - \vec{m}$. And draw a picture!]

6. Section 12.2 #51.

7. Section 12.2 #7.

8. In any given year, a company's *revenue* is its income — that is, the amount of money that it pulls in. (The profit is the revenue minus the cost.) Let's denote the revenue as r. Notice that r can change from year to year. Now consider an industry made up of two companies with revenues r_1 and r_2 respectively. The overall state of the system can be represented as a vector $\vec{r} = \langle r_1, r_2 \rangle$ pointing to the point (r_1, r_2) in the first quadrant of $\mathbb{R}^2$. Here are four questions.

   (a) If the two companies are competing for the same fixed pool of consumers, then how might $\vec{r}$ vary over time? Draw a picture and briefly explain.

   (b) On the other hand, if the two companies are in cooperation — for example, one supplies the other with materials — then how might $\vec{r}$ vary over time?

   (c) Re-do those first two questions for the case where there are three companies.

(d) What if there are n companies, where $n = 100$, say?

If you feel that you need more practice with this foundational vector material, then do more problems from the book, or come talk to me. :)