1. Section 12.4 #2.
2. Section 12.4 #3.
3. Section 12.4 #13.
4. Section 12.4 #16.
5. Section 12.4 #26.
6. (By the way, this problem is similar to #44.) Suppose that $\vec{v}$ and $\vec{w}$ are non-zero and perpendicular. Find a vector $\vec{x}$, in terms of $\vec{v}$ and $\vec{w}$, such that $\vec{v} \times \vec{x} = \vec{w}$. Also, why is it important that $\vec{v}$ and $\vec{w}$ be perpendicular? Also, why is it important that $\vec{v}$ and $\vec{w}$ be non-zero?
7. In robotics, astronomy, computer graphics, and other fields, we need to be able to compute how objects revolve around each other. To that end, let $\vec{u}$ be a unit vector in $\mathbb{R}^3$, let $\vec{v}$ be any vector in $\mathbb{R}^3$, let α be any angle, and let

$$\vec{w} = (\vec{u} \cdot \vec{v})(1 - \cos\alpha)\vec{u} + (\cos\alpha)\vec{v} + (\sin\alpha)\vec{u} \times \vec{v}.$$

   Imagine that $\vec{u}$ is an axis of rotation (like an axle on a car). Place $\vec{v}$ tail-to-tail with $\vec{u}$. Rotate $\vec{v}$ about $\vec{u}$ in a right-handed manner through the angle α. I claim that the resulting vector is the $\vec{w}$ defined above. Your job is to partially verify this claim. Here are four problems.

   (a) Check that $\vec{w} \cdot \vec{u} = \vec{v} \cdot \vec{u}$.

   (b) Check that $|\vec{w}| = |\vec{v}|$.

   (c) Draw a picture of what's going on.

   (d) Is there anything else that should be checked, to verify the claim? (I'm not asking you to check it. I'm just asking you to describe in English what needs to be checked, if anything.)

8. In $\mathbb{R}^3$, let $\vec{q}$ be a point, and let $\vec{x}(t) = \vec{p} + t\vec{d}$ be a parametrized line. I'd like you to find the point on the line that is closest to $\vec{q}$. In fact, I want you to solve this problem in two different ways.

   (a) Express the distance between $\vec{q}$ and $\vec{x}$ as a function of t, and minimize it. (Hint: The algebra is easier if you minimize the square of the distance, rather than the distance itself.)

(b) At the closest point $\vec{x}$, the line should be perpendicular to the line through $\vec{x}$ and $\vec{q}$. Use this geometric insight to solve the problem.

Homework constitutes the preceding eight questions. What follows is an optional ninth question. You are not required to hand it in. It is not worth extra credit. I provide it to you, just in case you want some more practice.

Planets, moons, comets, etc. tend to move in elliptical orbits. However, in this problem we are dealing with such a small segment of orbit, that we can reasonably approximate it as a straight line. Also, let's assume that our universe is two-dimensional (although the problem is not really any harder in three dimensions).

An astronomer working late at night notices a previously unknown asteroid on her computer. The asteroid is at position $\vec{p}$ and moving with velocity $\vec{v}$. (The unit of distance is 10^6 m, measured relative to the Sun, and the unit of time is the hour. Time $t = 0$ is when the astronomer discovers the asteroid.) At that moment, Earth is at position $\vec{q}$ and moving with velocity $\vec{w}$.

a) Write expressions, in terms of $\vec{p}$, $\vec{v}$, $\vec{q}$, $\vec{w}$, and t, for these three quantities: the position of the asteroid at time t, the position of the Earth at time t, and the distance between the two bodies at time t (regarding them as point particles, although they're really spherical).

b) Still working in terms of $\vec{p}$, $\vec{v}$, $\vec{q}$, $\vec{w}$, and t, find the time t at which the two bodies are closest, and how close they are at that time. (Hint: Don't minimize the distance. Rather, minimize the square of the distance. Also, you don't actually need any calculus.)

c) Suppose that $\vec{p} = (213268.00, 208956.00)$, $\vec{v} = \langle 56.66, 113.32\rangle$, $\vec{q} = (212132.00, 212132.00)$, and $\vec{w} = \langle 75.66, -75.66\rangle$. Earth's radius is 6.38 and the asteroid's radius is 0.1. Will the asteroid hit Earth?

d) Using the same values for $\vec{p}$, $\vec{v}$, $\vec{q}$, $\vec{w}$ as in the previous part, where will the asteroid be, 12 hours after its discovery by the astronomer? Where will Earth be, 24 hours after the discovery?