Section 15.4 #3, 17, 18, 29, 30.

Section 15.6 #2.

For the last problem of this assignment, recall that we used the Mathematica notebook integration.nb on Day 15 of the course. The notebook ends with the example of

$$\iint_R \frac{x-y}{x^3+y^3}\, dA,$$

where $R = [0,1] \times [0,1]$.

A. What does Mathematica give for the value of the integral, computed in the $dx\, dy$ order and in the $dy\, dx$ order?

B. By mimicking how I used the `integrate` (not `Integrate`!) function in earlier examples, compute a Riemann sum approximation to the integral. You might want to try a couple of values for `n`. Show the line(s) of code that you used and the result(s).

C. Explain how these results relate to Fubini's theorem.

What follows is a bonus problem. You are not required to hand it in, and it will not be graded, but it might help you study.

Richard Feynman (1918–1988) was one of the most successful (and colorful, and controversial) physicists of the 20th century. In his memoirs, he gleefully revealed his favorite mathematical trick: "differentiation under the integral sign". Basically, it amounts to pushing a differentiation with respect to one variable into an integral with respect to a different variable:

$$\frac{d}{dx}\int_a^b f(x,y)\, dy = \int_a^b \frac{\partial}{\partial x} f(x,y)\, dy.$$

That's the backstory. Now here's your problem. Let

$$I(a) = \int_0^\infty \frac{e^{-x} - e^{-ax}}{x}\, dx.$$

Show that $I(a) = \log a$ (meaning the natural logarithm).