Section 16.3 #3, 4, 5, 8. (Yes, I mean Section 16.3.)

Section 16.2 #1, 3, 11. (Yes, I mean Section 16.2.)

For this final, multi-part problem, let D be the set of points in the plane other than the origin:

$$D = \{(x, y) : x \neq 0 \text{ or } y \neq 0\}.$$

Also let

$$\vec{F}(x, y) = \langle P(x, y), Q(x, y) \rangle = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle.$$

Notice that $\vec{F}$ is defined on all of D.

A. Show that $\frac{\partial}{\partial y} P = \frac{\partial}{\partial x} Q$ everywhere on D.

B. Find all potential functions for $\vec{F}$ on the part of D where $x > 0$. Your answer should involve an unknown constant C_1. (Hint: Consider something like $\arctan(y/x)$ or $\arctan(-x/y)$.)

C. Find all potential functions for $\vec{F}$ on the part of D where $y > 0$. Your answer should involve a C_2.

D. Find all potential functions for $\vec{F}$ on the part of D where $x < 0$, using a C_3.

E. Find all potential functions for $\vec{F}$ on the part of D where $y < 0$, using a C_4.

F. Explain why there is no potential function for $\vec{F}$ on all of D. (Hint: If such a function existed, then what would its values be at the four points $(x, y) = (\pm 1, \pm 1)$?)