Section 16.4 #14, 15, 22.

For this final set of problems, consider the triangle in $\mathbb{R}^2$ whose vertices are $\vec{p} = (p_1, p_2)$, $\vec{q} = (q_1, q_2)$, and $\vec{r} = (r_1, r_2)$ in counterclockwise order. Let $\vec{F} = \langle -y/2, x/2 \rangle$.

A. Show that the area of the triangle is the line integral of $\vec{F}$ along the (counterclockwise) boundary.

B. Compute the line integral of $\vec{F}$ over the line segment from $\vec{p}$ to $\vec{q}$ (not from $\vec{q}$ to $\vec{p}$). Simplify your answer as much as possible.

C. Using problem A once and problem B three times, give a simple expression for the area of the triangle in terms of $p_1, p_2, q_1, q_2, r_1, r_2$.

That's it for the homework. What follows are some related problems, which you are not expected to hand in. They might help you study for Exam D. By the way, this material is not just a math exercise; I have seen it used to solve problems in geophysics and in computer graphics.

D. Re-derive the expression for area in problem C, more quickly, by using the cross product of certain vectors.

If problem D makes you feel that problems A–C were a waste of time, then problems E and F might make you feel better. They show that problems A–C generalize in a way that problem D does not.

E. Extrapolating from problem C, guess an expression for the area of an n-sided polygon, whose boundary is a simple closed curve, with vertices listed in counterclockwise order.

F. Returning to the triangle, it is possible to compute the moments M_y and M_x by following the same approach with a different $\vec{F}$?