You have 60 minutes. No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit. It is often a good idea to draw a picture.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Mark your final answer clearly. Good luck.

A1. Let $\vec{F} = \langle 3x^2 \sin y, x + yz, x + z^4 \rangle$. What is curl $\vec{F}$?

A2. What is div $\vec{F}$?

A3. What is div(curl $\vec{F}$)?

A4. Is $\vec{F}$ conservative?

B. A businessperson has a total of \$$d$ to invest in three businesses. She will invest \$$x$ in the first, \$$y$ in the second, and \$$z$ in the third. If life were simple, then her expected profit in the first year would be $ax + by + cz$, for some constants a, b, c. (For example, if the first business produced a 7% return, then a would be 0.07.) However, tax policy penalizes the first business when it is associated with the second and the second when associated with the third. So her expected profit is more like $(a - hy)x + (b - kz)y + cz$, where h and k are other constants.

B1. Use Lagrange multipliers to write the profit maximization problem as a system of equations.

B2. Solve the system of equations (in terms of the constants a, b, c, d, h, k).

C. The Earth is a ball of radius $R = 6371$ (in km). Its density increases with depth. At a distance d from the center of the Earth, the density is roughly $Ad^2/R^2 + B$ (in kg/km^3), where $A = -11 \cdot 10^{12}$ and $B = 14 \cdot 10^{12}$. Compute the mass of the Earth (in kg).

D. Let $(a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n)$ be n given points in the x-y-plane. For any point (x, y), let $f(x, y)$ be the sum of the squares of the distances from (x, y) to the points (a_i, b_i). Show that f achieves its smallest value when x is the average of the a_i and y is the average of the b_i.

E. Evaluate $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{4x^2 + 5y}\, dx\, dy$. (Hint: Change the order.)