You have 150 minutes. No notes, books, calculators, computers, etc. are allowed.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit. It is often a good idea to draw a picture.

You might find these calculus facts helpful:

$$\begin{aligned}
\frac{d}{dx}\arctan x &= \frac{1}{1+x^2}. \\
\frac{d}{dx}\left(\frac{1}{2}x - \frac{1}{4}\sin(2x)\right) &= \sin^2 x. \\
\frac{d}{dx}\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) &= \cos^2 x.
\end{aligned}$$

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic. Mark your final answer clearly. Good luck.

A1. Let P be the plane though the point $\vec{p}$ with normal vector $\vec{n}$. Let Q be the plane though the point $\vec{q}$ with normal $\vec{m}$. Let $\vec{s}$ be a point at which P and Q intersect. Parametrize the line, along which P and Q intersect. Give your answer in terms of $\vec{p}$, $\vec{n}$, $\vec{q}$, $\vec{m}$, and $\vec{s}$.

A2. In which special cases will your answer for Problem A1 not work? Explain.

B1. Let $\vec{F} = \langle e^{yz}, xe^{yz}z, xe^{yz}y \rangle$. Find a potential function for $\vec{F}$.

B2. Let C be parametrized by $\vec{r}(t) = (\cos t, t^2, \log(t+1))$ for $0 \leq t \leq 1$. Compute $\int_C \vec{F} \cdot d\vec{s}$.

C1. Find *all* of the critical points of $f(x, y) = x^3 + y^4 - 6x - 2y^2$.

C2. For any *one* of those points, determine whether it is a local max, min, saddle, etc.

D1. Let $\vec{F} = \langle -e^z, 1/(1+x^2), -2 \rangle$. Show that $\vec{F} = \operatorname{curl} \vec{G}$, where $\vec{G} = \langle 2y, e^z, -\arctan x \rangle$.

D2. Compute $\iint_S \vec{F} \cdot d\vec{S}$, where S is the part of the paraboloid $z = 4 - x^2 - y^2$ above the x-y-plane, with upward-pointing normals.

E1. Let W be the three-dimensional region that lies above the x-y-plane, lies inside the sphere of radius 3 centered at the origin, and is bounded by $x = 1$, $y = 0$, and $x = y$. Draw it.

E2. Compute the integral of $f(x, y, z) = z$ over W.

F. Optimize $2x + 5y$ on the ellipse $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$.

G. For any scalar function $f(x, y, z)$, define $\Delta f = f_{xx} + f_{yy} + f_{zz}$. Show that $\Delta f = \text{div}(\text{grad } f)$.

H. Guess the product rule that begins "$\text{div}(f\vec{F}) = \ldots$". Your answer should be sensible, with no operation applied illegally, but you do *not* need to check algebraically that it is correct.