You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

When you are asked to “prove” something, you are being asked to give a convincing explanation of why that fact is true. Imagine that your audience is a skeptical classmate.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do simple arithmetic, but don’t bother to do complicated arithmetic. Mark your final answer clearly.

Good luck. :)

A. Let $\vec{v} = \langle 1, 3, -2 \rangle$ and $\vec{w} = \langle 1, 1, 2 \rangle$. Calculate these quantities.

$\vec{v} + \vec{w} =$

$-5\vec{v} =$

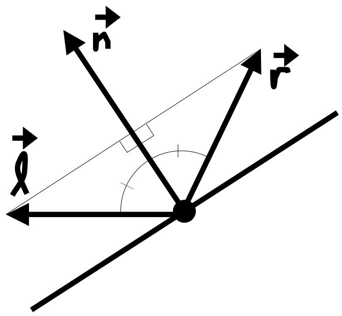
$\vec{v} \cdot \vec{w} =$

$\text{proj}_{\vec{w}}\vec{v} =$

$\vec{v} \times \vec{w} =$

B. Consider the line through the points $(1, 0)$ and $(0, 1)$ in $\mathbb{R}^2$. Give an equation for this line in polar coordinates, with r written as a function of θ.

C1. In computer graphics, a common operation is reflecting a vector $\vec{\ell}$ across a unit vector $\vec{n}$. See the picture below. Prove that the reflected vector $\vec{r}$ equals $2(\vec{n} \cdot \vec{\ell})\vec{n} - \vec{\ell}$.



C2. As a check on problem C1, compute the dot product of $2(\vec{n} \cdot \vec{\ell})\vec{n} - \vec{\ell}$ with itself, simplifying as much as possible. Interpret your answer geometrically. Does it agree with C1?

D. Let $ax + by + cz = d$ and $ex + fy + gz = h$ be two planes in $\mathbb{R}^3$. Assume that they are not parallel. Then they intersect in a line. Let $\vec{p}$ be a point on the line. Parametrize the line, giving your answer in terms of the numbers a, b, c, d, e, f, g, h, and/or the vector $\vec{p}$.

E. Suppose that the curve $\vec{x}(t) = \langle e^{2t}, \sqrt{t}, t\cos(t + \pi)\rangle$ describes the trajectory of a dolphin as it swims and frolics merrily. Find the acceleration of the dolphin at the point $\langle 1, 0, 0\rangle$.

F. In recent homework you derived the differential equation $|\vec{x}|^3\,\vec{x}\,'' = -GM\vec{x}$ for the trajectory of a planet in orbit about a star. Suppose that, at time $t = 0$, the planet is at position $\vec{p} = \vec{x}(0)$ and moving with velocity $\vec{d}$.

F1. What is the planet's acceleration at $t = 0$ (in terms of the information given)?

F2. Usually, when we are trying to solve a differential equation, we cannot hope to find an exact, symbolic solution. Instead we resign ourselves to an approximate numerical solution, which is based on the simplifying assumption that some derivative is constant over a small interval of time. In that spirit, let's assume that the planet's acceleration is constantly the value that you found in problem F1. What, then, is the planet's position at time t?