You have 150 minutes.

No books, calculators, computers, etc. are allowed. You are allowed a "crib sheet" that occupies both sides of a standard sheet of paper, made by you, not to be shared with anyone.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

In particular, if you do not have time to finish a problem, then be clear about how you've set the problem up, and what strategy you would pursue from that point, if you had more time.

Perform as much algebraic simplification as you can. Do simple arithmetic, but don't bother to do complicated arithmetic. Mark your final answer clearly.

Remember that I am grading your paper, not you. Make sure your paper says exactly what you want it to say.

Good luck. :)

A. Compute the divergence of $\vec{F} = \langle 2x + 3y - z, y + 2z, -x - 3z \rangle$.

B. Let $\vec{F} = \langle 6xy + 2, 3x^2 + 6y \rangle$. Find a potential function for $\vec{F}$, or show that none exists.

C. Consider the curve that traces out an equilateral triangle in the plane, starting at $(1, 0)$, then going to $(0, \sqrt{3})$, then to $(-1, 0)$, then back to $(1, 0)$. Compute the integral of the vector field $\vec{F} = \langle x\cos(x^2), xy \rangle$ along this curve.

D. A particle travels along a straight-line trajectory $\vec{x}$ in $\mathbb{R}^3$, from $\vec{x}(0) = \vec{p}$ to $\vec{x}(1) = \vec{q}$. Meanwhile, a magnetic field $\vec{B} : \mathbb{R}^3 \to \mathbb{R}^3$ permeates space. At any instant of time, the magnetic force on the particle is $k\vec{v} \times \vec{B}$, where $\vec{v}$ is the particle's velocity and k is a positive constant. Compute the work done by the magnetic force on the particle.

E. Consider the tetrahedron bounded by the planes $z = 0$, $y + z = 0$, $2x + y + 2z = 2$, and $-2x + y + 2z = 2$. By the way, its vertices are $(1, 0, 0)$, $(-1, 0, 0)$, $(0, 2, 0)$, and $(0, -2, 2)$. Set up, but do not compute, an iterated integral of a function $f(x, y, z)$ over the tetrahedron.

F. Guess the product rule that begins "curl$(f\vec{F}) = \ldots$", where $f : \mathbb{R}^3 \to \mathbb{R}$ is a scalar field and $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ is a vector field. Your answer should be sensible, with no operation applied illegally. Indicate briefly how you know that each operation is being applied legally. However, you do not need to prove that your answer is correct.

G. Consider the plane in $\mathbb{R}^3$ through the point $\vec{p}$ with normal vector $\vec{n}$. Explain, using words and pictures, why all points $\vec{x}$ in this plane satisfy the equation $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$, and why no other points $\vec{x}$ satisfy that equation.

H. A company makes two products: screens for phones, and complete phones. It must make at least as many screens as phones, because it uses its screens in its phones. If it makes s extra screens, then it can sell them to other companies for a profit of $\$80\sqrt{s}$. Meanwhile, making p phones results in a profit of $\$300\sqrt{p}$. Also, the company's supply of rare-earth elements limits its screen production to 1,000,000 screens total.

H.A. Formulate the problem of maximizing profit. Be precise about which function is being maximized and on which domain or subject to which constraints. Include a picture.

H.B. Solve the maximization problem. Do not skip steps; be explicit, so that I am sure that you understand how to solve all maximization problems of this kind. (Continue on the back?)