If f and g are differentiable functions, then the product rule says that

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x).$$

Let's anti-differentiate (meaning, compute the indefinite integral of) both sides:

$$\begin{aligned} (f(x)g(x))' &= f'(x)g(x) + f(x)g'(x) \\ \Rightarrow \quad \int (f(x)g(x))'\ dx &= \int f'(x)g(x) + f(x)g'(x)\ dx \\ \Rightarrow \quad f(x)g(x) + C &= \int f'(x)g(x)\ dx + \int f(x)g'(x)\ dx. \end{aligned}$$

This last equation is sometimes called a technique, and that technique is called *integration by parts*. Usually the equation is rewritten with the last term isolated, like this:

$$\int f(x)g'(x)\ dx = f(x)g(x) + C - \int f'(x)g(x)\ dx.$$

In fact, you can omit the "+ C", because there is still an anti-derivative to be computed on the right side of the equation. So we have

$$\int f(x)g'(x)\ dx = f(x)g(x) - \int f'(x)g(x)\ dx.$$

Here's an example. Suppose that we want to compute

$$\int x \sin x\ dx.$$

The key insight is that the integrand is a product of two factors, one of which (x) gets simpler when we differentiate it, and the other of which ($\sin x$) is easy to anti-differentiate. Let $f(x) = x$ be the thing whose derivative is simpler, and let $g'(x) = \sin x$ be the thing that's easy to anti-differentiate. Then $f'(x) = 1$ and $g(x) = -\cos x$. (You can omit the "+ C" here.) In this notation, we're trying to compute

$$\int f(x)g'(x)\ dx.$$

So integration by parts tells us that the answer is

$$f(x)g(x) - \int f'(x)g(x)\ dx = -x\cos x - \int -\cos x\ dx.$$

We still have to do another anti-differentiation problem, but the point is that this new problem is easier than the old one. We quickly obtain a final answer of

$$\int x \sin x\ dx = -x\cos x + \sin x + C.$$

It's a good idea to check that the answer is correct by differentiating it. When you do so, you use the product rule. That's because integration by parts is just the product rule.

Here are some exercises for you to try. Notice that in each example the integrand is a product of two factors. One will be $f(x)$ and one will be $g'(x)$. You might have to try both possibilities.

1. What is $\int e^x x \, dx$?
2. What is $\int x^7 \log x \, dx$? (Here, "log" denotes the natural log — that is, $\log_e$.)
3. What is $\int x^2 \sin x$? (Hint: You have to do parts twice.)
4. In the worked example, I claimed that you don't need to append "$+ C$" when you compute $g(x)$ from $g'(x)$. Why is that true?

For more explanation and exercises, consult a calculus textbook.