Much of Exam D is about Sections 16.1–16.5: line integrals, Green's theorem, gradient, curl, and divergence. But some of Exam D is about earlier material. To help you review that material, here are some review problems. Do whichever ones you like.

A. Let $\vec{p}, \vec{q}$ be points. Parametrize the line through them. Actually, come up with two parametrizations $\vec{x}(t)$. In the first one, set things up so that $\vec{x}(0) = \vec{p}$ and $x(1) = \vec{q}$. In the second one, set things up so that $\vec{x}$ travels with constant speed 1, and give the values of t where $\vec{x}$ hits $\vec{p}$ and $\vec{q}$.

B. Let $\vec{p}, \vec{q}, \vec{r}$ be three points in $\mathbb{R}^3$. Write an equation for the plane that contains them. Are there any special cases, where your equation does not work? What is the geometric interpretation of those special cases?

C. At time $t = 0$, a particle is at a certain position $\vec{p}$ and is moving with a certain velocity $\vec{v}$. Suppose that the acceleration $\vec{a}$ of the particle is constant. Find an expression for the position $\vec{x}(t)$ of the particle at all times t, in terms of $\vec{p}, \vec{v}, \vec{a}$.

D. Let a and b be positive constants. Consider the ellipse $x^2/a^2 + y^2/b^2 = 1$ in $\mathbb{R}^2$. Parametrize this ellipse. Write, but do not compute, an integral that expresses the arc length of the ellipse. (The integral itself is famously difficult.)

E. Let $\vec{x} : \mathbb{R} \to \mathbb{R}^4$ be a parametrized curve. That is, $\vec{x}(t) = \langle x_1(t), x_2(t), x_3(t), x_4(t) \rangle$. Define $\vec{a}(t) = \langle -x_2'(t), x_1'(t), -x_4'(t), x_3'(t) \rangle$. Show that $\vec{a}$ is always perpendicular to the velocity of $\vec{x}$. Then find two other functions, $\vec{b}(t)$ and $\vec{c}(t)$, that are also perpendicular to the velocity. There is a "cheating" way to do this, by simply making $\vec{b}$ and $\vec{c}$ scalar multiples of $\vec{a}$. Avoid that. Find a solution such that (except maybe at special values of t) no two of $\vec{a}, \vec{b}, \vec{c}$ are parallel to each other.

F. Consider the monkey saddle $z = x^3 - 3xy^2$. Find an equation for the tangent plane at the point $(1, 2, -11)$.

G. As part of your research into climate change, you want to study how water temperature varies along the Gulf Stream (which is a huge oceanic current along the east coast of North America). For simplicity, approximate the Atlantic Ocean as a flat plane with coordinates x, y. Let $T(x, y, t)$ denote the temperature of the water at position (x, y) on the surface at time t. In English, why does it make sense that T could depend on x and y? Why does it make sense that T could depend on t?

Anyway, you build a scientific instrument by taping a battery, an electronic thermometer, and a radio transmitter to a buoy. You drop the instrument into the ocean near Florida, and let it float northeast. Once per minute, the instrument transmits its measurement of the water temperature. Back at your home base, you receive these temperature measurements, watching them change slightly from minute to minute. Use the chain rule to explain in detail how temperature is changing with respect to time in this situation.

H. You are designing a new kind of energy-efficient building. The building will be in the shape of a rectangular box, with walls running east-west and north-south. The exterior of the building will be covered with a photo-voltaic material, which produces electricity from sunlight. Because of how sunlight hits the building, the top of the building will generate 1.8 power units (specifically, kWh per day) per m^2, the south side 0.9 units per m^2, the east and west sides 0.2 units per m^2, and the north side nearly 0 units per m^2. Each wall of the building must be at least 10 m long and at least 3 m tall. The volume of the building must be exactly 3600 m^3.

Given those specifications, you'd like to maximize the energy collection of the building. Formulate this problem as an optimization problem, explicitly: Which function is being optimized, and over which domain or subject to which constraints? Then solve that optimization problem.

Given those same specifications, you'd like to minimize the surface area where the building is in contact with the air, because that is where the building loses heat in the winter and coolness in the summer. Keep in mind that the bottom of the building is not in contact with the air. Formulate, but do not solve, this optimization problem.

Finally, here's an open-ended question with no single correct answer: What more information do you need, to combine the two optimization problems into a single problem, whose solution is the best possible building overall? Formulate, but do not solve, that optimization problem.

I. In the plane, draw two circles of radius R, whose centers are distance R from each other. Thus each circle passes through the other circle's center. Let D be the region enclosed by one circle but outside the other circle. Using polar integration, show that the area of D is $R^2(\pi/3 + \sqrt{3}/2)$. It might help to know that $\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$.

J. Consider the tetrahedron bounded by the planes $z = 0$, $x + z = 0$, $x + 2y + 2z = 2$, and $x - 2y + 2z = 2$. By the way, its vertices are $(2, 0, 0)$, $(0, 1, 0)$, $(0, -1, 0)$, and $(-2, 0, 2)$. Set up, but do not compute, an integral of a function $f(x, y, z)$ over the tetrahedron. While you're at it, why not do this problem five more times, using the other five possible orders of integration?

K. Compute the distance along the Earth's surface from Jakarta (6.20° S, 106.82° E) to Moscow (55.75° N, 37.57° E). By the way, the radius of the Earth is 6,371 km.

L. Show that the curl of a (sufficiently differentiable) conservative three-dimensional vector field is everywhere $\vec{0}$.