A.A. The sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

A.B. An example outcome is 3. Another example outcome is 2.

A.C. An example event is $\{6, 1, 4\}$. Another example event is $\{4\}$.

A.D. The distribution of X is the discrete uniform distribution on $\{1, 2, 3, 4, 5, 6\}$.

A.E. The PMF of X is $P(X = k) = 1/6$ for $k = 1, 2, \ldots, 6$.

A.F. The PMF of $X + X$ is $P(X + X = k) = 1/6$ for $k = 2, 4, \ldots, 12$.

B. Let R be the event that I roll the rigged die, and let T be the event that I roll three 6s. We are told that $P(R) = 1/10$ and so $P(R^c) = 9/10$. Also, $P(T|R) = 0.28^3$ and $P(T|R^c) = 1/6^3$. Using Bayes' theorem and the law of total probability in the usual way, we compute

$$\begin{aligned} P(R|T) &= \frac{P(T|R)P(R)}{P(T|R)P(R) + P(T|R^c)P(R^c)} \\ &= \frac{0.28^3 \cdot 1/10}{0.28^3 \cdot 1/10 + 1/6^3 \cdot 9/10}. \end{aligned}$$

[This number is approximately 34%.]

C.A. FALSE. [This is an example from our textbook. $P(AB) = P(\emptyset) = 0 \neq P(A)P(B)$.]

C.B. TRUE. [This is one of the equivalent definitions of independence of two events. The logic is $P(AB) = P(A|B)P(B) = P(A)P(B)$.]

C.C. TRUE. [To construct an example, declare that $P(A) = P(B) = P(C) = 1/4$. Then the given inequality is equivalent to $P(AB) > P(BC) > P(CA)$. It is easy to satisfy this inequality by making A intersect C very little and making A intersect B a lot.]

C.D. FALSE. [There are $\binom{52}{5}$ hands.]

C.E. TRUE. [The counting process goes: Choose a rank for the three-of-a-kind. Then choose three cards in that rank. Then choose a rank for the pair. Then choose two cards in that rank. By the way, it works out to 3,744 hands.]

D.A. If I sample with replacement (for example, I release frogs after I've measured them, and I don't mark them in any way), then how many frogs must I examine, before I find my first infected one? The parameter is $p = F/(N + F)$.

D.B. If I sample without replacement (for example, I don't release frogs after I've measured them, or I mark them and don't examine frogs already marked), then how many infected frogs will I find in my first 100 examinations? The parameters are F, N, and $n = 100$, in that order.

D.C. If I sample with replacement, then how many infected frogs will I find in my first 100 examinations? The parameters are $n = 100$ and $p = F/(N + F)$.

D.D. If I sample with replacement, then how many frogs must I examine, before I find 100 infected frogs? The parameters are $r = 100$ and $p = F/(N + F)$.

E. [I made a mistake when writing this problem. So my intended solution is not correct, which means that the difficulty of the problem is not what I expected. To compensate for my mistake, I graded generously.]

[Here is the intended, incorrect solution: It's a Bose-Einstein-style problem. The 54 cups are indistinguishable objects, and the 18 faculty are distinguishable boxes. There are

$$\binom{54+18-1}{18-1} = \binom{71}{17} = \frac{71!}{17!54!}$$

ways for the cups to be assigned. They are equally probable, because each morning's assignment is independent of the others and uniformly random. In only one of those ways do all 18 faculty end up with 3 cups each. So the answer is

$$\frac{1}{\binom{71}{17}} = \frac{17!54!}{71!}.$$

This number is approximately 10^{-16}. Anyway, what is wrong with this solution? And now for the better solution...]

The faculty are distinguishable. The cups of coffee are indistinguishable, but we will treat them as distinguishable and then correct for overcounting. There are 18^{54} ways to assign the cups to the faculty, because there are 18 options per cup and 54 cups. Now how many of those 18^{54} ways result in three cups per faculty member? Picking an assignment that assigns three cups to each faculty member amounts to choosing three cups for the first faculty, three for the second, three for the third, and so on. So there are

$$\binom{54}{3}\binom{51}{3}\cdots\binom{6}{3}\binom{3}{3} = \frac{54!}{51!3!}\frac{51!}{48!3!}\cdots\frac{6!}{3!3!}\frac{3!}{0!3!} = \frac{54!}{(3!)^{18}}$$

ways. So the probability is

$$\frac{54!}{6^{18}18^{54}}.$$

[This number is approximately 10^{-11}. So it is much larger than the incorrect answer above. Why does that make sense?]

[Also, for the sake of historical accuracy, let me add: I accidentally counted myself among the 18 faculty. I should have written the problem with 17 other faculty, the 52nd day, etc. But that is not a mathematical error, at least.]