You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

On TRUE-FALSE questions there are four valid answers. If the correct answer is TRUE, then TRUE earns 3 points, TRUISH earns 2 points, FALSISH earns 1 point, and FALSE earns 0 points. If the correct answer is FALSE, then these point values are of course reversed. Do not write just T or F; write your answer completely and clearly. No explanation is needed.

On all other problems, show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Define any notation that you introduce. For example, if you write "$P(A)$", but neither I nor you have defined an event A, then that writing is not good.

Perform as much algebraic simplification as you can. Do not bother to do non-trivial arithmetic unless it is specifically requested. Mark your final answer clearly.

Good luck.

A.A. Let $X \sim \text{Expo}(\lambda)$. Then $E(X^4)$ can be computed using the law of the unconscious statistician. Set up the integral in detail, but do not compute its numerical value.

A.B. Let $Y = X^4$. Then $E(Y)$ can be computed using f_Y. Figure out f_Y and set up the integral for $E(Y)$ in detail, but do not compute the integral's numerical value.

A.C. Are the two integrals equal? Answer thoroughly.

B.A. Let $X_1, \ldots, X_n \sim \text{Pois}(\lambda)$ be IID. Let $S_n = X_1 + \cdots + X_n$. If $E(S_n)$ can be computed, then compute it; if not, then explain why not.

B.B. If $\text{Var}(S_n)$ can be computed, then compute it; if not, then explain why not.

B.C. Now suppose $X_1, \ldots, X_n \sim \text{Pois}(\lambda)$ and, for $i \neq j$, $E(X_i X_j) - E(X_i)E(X_j) = \lambda/2$. Do your answers to B.A and B.B change? If so, how?

C. TRUE or FALSE: For all jointly distributed continuous random variables X and Y, $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\, dy$.

D. Suppose that a, b, and c are constants and $a > 0$. What is $\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)}\,dx$? (Hint: $ax^2 + bx + c = \left(\sqrt{a}x + \frac{b}{2\sqrt{a}}\right)^2 - \left(\frac{b}{2\sqrt{a}}\right)^2 + c$. Try to avoid calculus.)

E. In the year 2232, a particle physicist is trying to measure Titus's constant, which according to theory is a positive number that controls how zombies decay into gh-gh-gh-ghosts. She has run her experiment many times, with varying results. In fact, 2.5% of the results are negative. Based on how her apparatus is designed, she believes that the results should be normally distributed about the true answer with variance $4 \cdot 10^{-6}$. What's the true answer?

F. You work for an online retailer that individually tracks its customers. Each customer is assigned a number λ, such that the occasions of her or his purchases follow a Poisson process with rate λ. For example, one of your customers is a misunderstood wizard named Voldemort. He hasn't got much of a nose, so he can't smell food, but he buys a lot of candy because he loves its sweet taste. He averages 5 purchases per week.

F.A. What is Voldemort's λ? Include everything I need to know, to evaluate your answer.

F.B. What is the expected time until Voldemort's next purchase?

F.C. What is the probability that Voldemort will make 3 or more purchases in the next 7 days?

F.D. How many purchases do you expect from Voldemort in the next 30 days?

G. TRUE or FALSE: If $Y = g(X)$, then $\mathrm{Cov}(X, Y) \neq 0$.

H. Shopping for pumpkins always reminds me of the Rayleigh distribution, which has PDF $xe^{-x^2/2}$ on support $(0, \infty)$. Suppose that I pick a value x from a Rayleigh X, and then you pick a value y, exponentially distributed with rate $\lambda = x^2$. Find $f_Y(y)$, including its support.