

A.A. Choose one adult. Let X be the number of Republicans you choose. Then $X \sim \text{Bern}(R/(R + D + I))$.

A.B. Repeatedly choose adults with replacement. Let X be the number of Republicans and independents chosen before you choose your first Democrat. Then $X \sim \text{Geom}(D/(R + D + I))$.

A.C. Choose n adults with replacement. Let X be the number of independents you choose. Then $X \sim \text{Binom}(n, I/(R + D + I))$.

A.D. Choose n adults without replacement. Let X be the number of independents you choose. Then $X \sim \text{HGeom}(I, R + D, n)$.

A.E. Repeatedly choose adults with replacement. Let X be the number of Republicans and independents chosen before you choose your eighth Democrat. Then $X \sim \text{NBinom}(8, D/(R + D + I))$.

B.A. Let F be the event that you observe a flapjack and B the event that the experiment produced a blorgon. We want

$$\begin{aligned} P(B|F) &= \frac{P(F|B)P(B)}{P(F|B)P(B) + P(F|B^c)P(B^c)} \\ &= \frac{2/3 \cdot 1/100}{2/3 \cdot 1/100 + 1/3 \cdot 99/100} \\ &= 2/101, \end{aligned}$$

which is about 2%.

B.B. Assume that the trials are independent. (If they're not, then we have nowhere near enough information to proceed.) Continuing the same notation as above, we want $P(FB \text{ before } FB^c)$. Because FB and FB^c are disjoint, we can use our standard result and then the definition of conditional probability:

$$\begin{aligned} P(FB \text{ before } FB^c) &= \frac{P(FB)}{P(FB) + P(FB^c)} \\ &= \frac{P(F|B)P(B)}{P(F|B)P(B) + P(F|B^c)P(B^c)}. \end{aligned}$$

Coincidentally, this part of the problem has the same answer as the previous part.

C. [This problem is a mutated version of Section 3.12 Exercise #8. A student, who had studied that problem, would probably do very well on this problem.] The support of X is $\{n, n + 1, \dots, 100\}$. How can it happen that $X = k$? Only when one of the boxes has value k and the

other $n - 1$ boxes have values less than k . Therefore

$$P(X = k) = \frac{\binom{k-1}{n-1}}{\binom{100}{n}}.$$

D.A. Your drive is a sequence of $E + N$ blocks, N of which go north and E of which go east. All that distinguishes one way from another is where the E east-going blocks are placed in the sequence of $E + N$ blocks. The answer is $\binom{E+N}{E}$ or, equivalently, $\binom{E+N}{N}$.

D.B. Your drive is a sequence of T blocks, in which each block has the option of being east-, west-, north-, or south-going, independently of the others. So the answer is 4^T .

D.C. [This problem is a bit ugly, but it hits issues that we have studied, such as overcounting. I put it last on the exam, in the hopes that students would prioritize other problems.]

Let's express intersections using coordinates (e, n) , where e and n are integers and the starting location is at $(0, 0)$. We are going to analyze which of these intersections are achievable (as the ending point of a drive exactly T blocks long) through a few cases.

First, because T is even, the origin $(0, 0)$ is achievable through a drive such as east-west-east-west-...-east-west. That is, we can "waste" the blocks in $T/2$ canceling east-west pairs.

Second, consider the "east axis" consisting of locations $(e, 0)$ where $e \geq 1$. The location $(T, 0)$ is achievable. So is the location $(T - 2, 0)$, because we can waste two blocks by introducing a canceling pair. Similarly, the locations $(T - 4, 0), \dots, (2, 0)$ are achievable. In total, exactly $T/2$ locations on the east axis are achievable.

Third, consider the "northeast quadrant" consisting of locations (e, n) where $e \geq 1$ and $n \geq 1$. If all T blocks are east- and north-going, then there are $T - 1$ achievable locations. But we can also introduce up to $T/2 - 1$ cancelling pairs. So, letting k represent the number of canceling pairs, the number of achievable locations is actually

$$\begin{aligned} \sum_{k=0}^{T/2-1} T - 2k - 1 &= \sum_{k=0}^{T/2-1} (T - 1) - 2 \sum_{k=0}^{T/2-1} k \\ &= T/2 \cdot (T - 1) - 2(T/2 - 1)(T/2)/2 \\ &= T^2/4. \end{aligned}$$

Finally, there are four symmetric copies of the second case and four symmetric copies of the third case. So the total number of achievable locations is

$$1 + 4 \cdot T/2 + 4 \cdot T^2/4 = 1 + 2T + T^2 = (T + 1)^2.$$

[I have explicitly checked that this answer is correct for $T = 0, 2, 4$. So I'm reassured.]

D.D. Bonus question: How would you craft a version of this problem, whose answer is $\binom{T+4-1}{4-1}$ by Bose-Einstein? Also: How do the answers to parts B and C change if T is odd?