A. You cover v sites if and only if the volunteers all go to distinct sites. So this is the birthday problem, with sites playing the role of birthdays (and people playing the role of people). The probability of the volunteers going to distinct sites is

$$\frac{s \cdot (s-1) \cdot \cdots \cdot (s-v+1)}{s \cdot s \cdot \cdots \cdot s} = \prod_{i=0}^{v-1} \frac{s-i}{s} = \prod_{i=0}^{v-1} \left(1 - \frac{i}{s}\right).$$

B.A. Each day of a hard drive's life is a Bernoulli trial with probability $p = 0.001$ of success (where "success" means breaking). Assuming that the trials are independent of each other, the number of breakdowns today is $X \sim \text{Binom}(100{,}000, 0.001)$.

B.B. The answer is

$$\begin{aligned} P(X > 100) &= \sum_{k=101}^{100{,}000} P(X = k) \\ &= \sum_{k=101}^{100{,}000} \binom{100{,}000}{k} (0.001)^k (0.999)^{100{,}000-k}. \end{aligned}$$

Another way to express the answer, which dovetails better with R, is

$$\begin{aligned} P(X > 100) &= 1 - P(X \leq 100) \\ &= 1 - \texttt{pbinom(100, 100000, 0.001)} \\ &= 0.4734378. \end{aligned}$$

B.C. Well, $Y \sim \text{Binom}(100{,}000, 0.001)$ too. So $X + Y$ can take on any integer value between 0 (when $X = 0$ and $Y = 0$) and 200,000 (when $X = 100{,}000$ and $Y = 100{,}000$). [As one student pointed out, Y should be a constant, in that we should know its value, because it counts failures that have already happened. That was not the intent of the problem. The problem would be clearer if Y were the number of hard drives failing tomorrow.]

B.D. [I give two solutions. This first solution mimics what we did in Homework 06 Problem C.] For each k in the support of $X + Y$, we need to consider all of the ways that $X + Y$ might equal k. For example, X could equal 0 and Y could equal k, or X could equal 1 and Y could equal $k - 1$, and so on. We find that

$$P(X + Y = k) = \sum_{i=0}^{k} P(X = i, Y = k - i).$$

Assuming that X and Y are independent, we can split each term to obtain

$$\sum_{i=0}^{k} P(X = i) P(Y = k - i) = \sum_{i=0}^{k} \binom{100{,}000}{i} (0.001)^i (0.999)^{100{,}000-i} \binom{100{,}000}{k-i} (0.001)^{k-i} (0.999)^{100{,}000-k+i}.$$

[This second solution exploits our understanding of what binomial random variables mean. You may or may not enjoy proving algebraically that it gives the same answer.] Intuitively, $X+Y$ counts the number of successes in two days worth of trials. There are 200,000 trials, and they're all independent. So $X+Y \sim \text{Binom}(200{,}000, 0.001)$, and

$$P(X+Y=k) = \binom{200{,}000}{k}(0.001)^k(0.999)^{200{,}000-k}.$$

C. Using Bayes's theorem and the law of total probability, we compute

$$\begin{aligned} P(H_3|D) &= \frac{P(D|H_3)P(H_3)}{P(D)} \\ &= \frac{P(D|H_3)P(H_3)}{P(D|H_1)P(H_1)+P(D|H_2)P(H_2)+P(D|H_3)P(H_3)} \\ &= \frac{0.11\cdot 0.1}{0.03\cdot 0.6+0.01\cdot 0.3+0.11\cdot 0.1} \\ &= 0.34375. \end{aligned}$$

D. The students are distinguishable, but we care only about the counts — not about which student is where. This is a Bose-Einstein problem. We are putting 80 balls into 4 boxes. The number of allocations is

$$\binom{80+4-1}{4-1} = \frac{83\cdot 82\cdot 81}{3\cdot 2\cdot 1} = 91{,}881.$$