You are expected to complete this exam during any two-hour window within the exam period, which runs from Wednesday 11:30 AM to Thursday 11:30 AM. The exam is open-book and open-note:

- You may use all of this course's resources: the textbook, your class notes, your old homework, and the course web site. You may not share any resources with any other person while you are taking the exam.
- You may cite material (definitions, theorems, examples, etc.) from class, the assigned textbook readings, and the assigned homework problems. You do not have to redevelop or reprove that material. On the other hand, you may not cite results that we have not studied.
- You may not consult any other books, papers, Internet sites, etc. You may use a computer for viewing the course Moodle/web site, running R commands, typing up your answers, and e-mailing with me. If you want to use a computer for other purposes, then check with me first.
- You may not discuss the exam in any way — spoken, written, etc. — with anyone but me, until everyone has handed in the exam.

Feel free to ask clarifying questions in person or over e-mail. You should certainly ask for clarification if you believe that a problem is mis-stated. If you cannot receive clarification, then explain your interpretation in your solution.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. You might want to work first on scratch paper and then recopy your solutions. Alternatively, you might want to type your solutions. Always show enough work and justification so that a typical classmate could understand your solutions. If you cannot solve a problem, then write a brief summary of the approaches you've tried. Partial credit is often awarded. Present your solutions in the order assigned.

Good luck. :)

In this first problem, let's agree that our unit of time is minutes. People arrive at a certain bus stop according to a Poisson process with rate λ. The bus arrives four times an hour, at exactly 15-minute intervals, and picks up everyone who is waiting.

A.A. Let X be the number of people who get on the bus at 8:15 AM on Friday 2021 February 26. What is the distribution of X?
A.B. What is the average value of X?
A.C. Working in this bus stop context, ask a question, whose answer is a continuous random variable, and state the distribution of that variable.
A.D. We have made a mathematical model of how the bus stop behaves. But no mathematical model is perfectly realistic. Identify at least two practical issues, that detract from this model's realism.

For our second problem, consider the following game. Each time you play the game, you roll a die, and you earn however many dollars the die shows. For example, you win \$4 if you roll a 4. The die in question is not fair. The probabilities of rolling 1, 2, 3, 4, 5, and 6 are respectively $p_1, p_2, p_3, p_4, p_5, p_6$, where $\sum p_i = 1$. State your answers in terms of the p_i.

B.A. Let X denote your winnings from a single play of this game. What is $V(X)$?
B.B. Suppose that you play the game n times, where n is some large number, for total winnings of T. What is $V(T)$?
B.C. The central limit theorem asserts that T is approximately normal. Taking that fact for granted, what are the parameters of this normal distribution?
B.D. A "realist" assumes that they are going to experience average luck. How much would a realist be willing to pay, to play this game n times (assuming that they knew the probabilities p_i)?
B.E. [This question is more difficult, and worth more points, than other questions. Remember that partial credit exists.] A "pessimist" is more conservative than a realist, because they worry that they will experience worse-than-average luck. In as much detail as possible, give an example of how a pessimist could use some of the information above, to determine how much they would be willing to pay, to play the game n times?

For our final problem, suppose that Xerxes picks a number $X \sim \text{Expo}(\lambda)$, and then his friend Yolande picks a number $Y \sim \text{Expo}(X)$.

C.A. What is the PDF of Y? Be as specific as possible, without undergoing a big calculation.
C.B. Do the big calculation. [State your method or strategy clearly, so that you can earn partial credit, even if you don't reach the correct final answer.]