A.A. By the assumptions of the problem, the people who get on the bus at 8:15 are the people who arrived between 8:00 and 8:15. By the definition of Poisson process, the number of arrivals in any 15-minute time period is $X \sim \text{Pois}(15\lambda)$.

A.B. The expectation is $E(X) = 15\lambda$.

A.C. Let Y be the time (in minutes) between one person's arrival and the next person's arrival. Then $Y \sim \text{Expo}(\lambda)$.

A.D. One issue is that the support of X is $\{0, 1, 2, 3, \ldots\}$. So it is possible that the bus will have to pick up more people than the bus can hold. A second issue is that the arrival times of people might not be uniformly distributed over each 15-minute interval. For example, people might walk to the bus stop together (causing clumping) or they might socially distance as they walk (causing whatever the opposite of clumping is). For another example, people might learn to time their arrivals just before the bus comes, because it's so predictable. A third issue is that no bus is so predictable.

B.A. First, the expectation is $E(X) = p_1 + 2p_2 + \cdots + 6p_6$. Next, we can compute $E(X^2) = p_1 + 4p_2 + \cdots + 36p_6$. Therefore

$$V(X) = E(X^2) - (E(X))^2 = (p_1 + 4p_2 + \cdots + 36p_6) - (p_1 + 2p_2 + \cdots + 6p_6)^2.$$

B.B. Assuming that the trials $X_1, \ldots, X_n$ are independent,

$$V(T) = V\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} V(X_i) = nV(X),$$

where $V(X)$ is as above.

B.C. In addition to what we've learned above, we need to know that $E(T) = nE(X)$. Then

$$T \sim \text{Norm}(E(T), V(T)) = \text{Norm}(nE(X), nV(X))$$

(where $E(X)$ and $V(X)$ are as above).

B.D. A realist expects to earn $E(X)$ on each play of the game. So the realist is willing to pay $nE(X)$ to play the game n times.

B.E. Suppose that the pessimist in question worries that they will be luckier than just 2.5% of players and unluckier than the other 97.5% of players. The amount that they are willing to pay is the number t such that $P(T \leq t) = 0.025$. By Theorem 5.4.5, this t is approximately two standard deviations less than the mean. So

$$t \approx nE(X) - 2\sqrt{nV(X)}.$$

C.A. First, we know that $f_X(x) = \lambda e^{-\lambda x}$ for $x > 0$ and $f_{Y|X}(y|x) = xe^{-xy}$ for $y > 0$. Using the definition of the marginal and conditional distributions (or the law of total probability), we compute

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \int_{-\infty}^{\infty} f_{Y|X}(y|x) f_X(x)\,dx.$$

Now there are two cases. If $y \leq 0$, then the integrand is 0, so $f_Y(y) = 0$. Henceforth assume $y > 0$. Then we obtain

$$\int_0^{\infty} xe^{-xy}\lambda e^{-\lambda x}\,dx = \int_0^{\infty} \lambda x e^{-x(y+\lambda)}\,dx.$$

C.B. Integration by parts (with $u = \lambda x$ and $dv = e^{-x(y+\lambda)}\,dx$) yields

$$\begin{aligned}
\int_0^{\infty} \lambda x e^{-x(y+\lambda)}\,dx &= \left[\frac{-\lambda x}{y+\lambda} e^{-x(y+\lambda)}\right]_0^{\infty} - \int_0^{\infty} \frac{-\lambda}{y+\lambda} e^{-x(y+\lambda)}\,dx \\
&= \left[\frac{-\lambda x}{y+\lambda} e^{-x(y+\lambda)}\right]_0^{\infty} - \left[\frac{\lambda}{(y+\lambda)^2} e^{-x(y+\lambda)}\right]_0^{\infty} \\
&= \left[e^{-x(y+\lambda)}\left(\frac{-\lambda x}{y+\lambda} - \frac{\lambda}{(y+\lambda)^2}\right)\right]_0^{\infty} \\
&= 0 - 1 \cdot \left(0 - \frac{\lambda}{(y+\lambda)^2}\right) \\
&= \frac{\lambda}{(y+\lambda)^2}.
\end{aligned}$$

In conclusion,

$$f_Y(y) = \begin{cases} \frac{\lambda}{(y+\lambda)^2} & \text{if } y > 0, \\ 0 & \text{if } y \leq 0. \end{cases}$$