There are four problems, labeled A-D, to be handed in on paper.

Whenever you store data on a computer or transmit it between computers, there is a chance that disturbances in the hardware will introduce errors into your data. To handle this issue, there are error-correction protocols operating invisibly everywhere, like the Force but more interesting. Hamming distance is an important concept in error correction. The *Hamming distance* $H(w, x)$ between two bit strings w and x is defined as follows. If $|w| \neq |x|$, then $H(w, x) = \infty$. If $|w| = |x|$, then $H(w, x)$ is the number of bits in which w and x differ. For example, $H(00010, 10111) = 3$. For any set A of bit strings, define $N_2(A)$ to be the set of bit strings within Hamming distance 2 of A:

$$N_2(A) = \{w : \exists x \in A \text{ such that } H(w, x) \leq 2\}.$$

A. Prove that if $A \subseteq \{0, 1\}^*$ is regular, then so is $N_2(A)$. (Hint: If $A = L(M)$, where M has states Q, then construct an NFA with states $Q \times \{0, 1, 2\}$.)

B. Working over $\Sigma = \{a, b, c, d\}$, draw an NFA that recognizes the same language as the regular expression

$$(ad \cup b \cup c)^*(dda)^* \cup ac.$$

Use the algorithm described in class and in our textbook (rather than, say, guessing).

C. Textbook regular expression syntax leaves out the complementation operation, even though regular languages are closed under complementation. Devise an algorithm for implementing complementation using the other regular expression operators. That is, your algorithm should input a regular expression R, and it should output a new regular expression S, such that $L(S) = \Sigma^* - L(R)$. (Probably your algorithm consists of algorithms, that we've discussed in class, tied together.) Illustrate your algorithm using the example of $R = a \cup b$ over the alphabet $\Sigma = \{a, b, c\}$.

If we didn't have time to do a pumping lemma example (such as $A = \{0^m 1^m : m \geq 0\}$) in class, then ignore this last problem; I'll put it on the next assignment. But if we did do an example, then do this problem. In either case, we'll do more pumping lemma problems on the next assignment.

Update: We did not finish the first example, so this problem is not part of this homework.

D. Do problem 1.29b (about www).