This first problem is like Problem 5.31 but easier. To understand it, first read the description of the $3x+1$ problem in Problem 5.31. It's a notorious unsolved problem in mathematics. Then, fix the input alphabet $\Sigma = \{0, 1\}$, and let

$$ALL_{TM} = \{\langle M \rangle : M \text{ accepts all strings over } \Sigma\}.$$

A. Suppose that you had a decider H for ALL_{TM}. Explain how you could use H to design a decider D that outputs the answer to the $3x + 1$ problem.

Recall that, for any two languages A and B,

$$A/B = \{w : \exists x \in B \text{ such that } wx \in A\}.$$

In an earlier assignment, we showed that if A is regular then A/B is regular. Our proof was non-constructive. This next problem suggests that the proof *must* be non-constructive — because it must be non-constructive even in the relatively tame case where B is recognizable.

B. Prove that such an N cannot exist. (Hint: Use some of the knowledge about A/B, which you toiled to gain in earlier assignments.)

This last problem is noteworthy, in that it's similar to Problem 5.15 but reaches the opposite conclusion.

C. Problem 5.14 (about a Turing machine that tries to move off the end of its tape).