Recall that a *property of recognizable languages* is a set A of Turing machine encodings $\langle M \rangle$ such that, for any two Turing machines M and N with $L(M) = L(N)$, either $\langle M \rangle$ and $\langle N \rangle$ are both elements of A or neither $\langle M \rangle$ nor $\langle N \rangle$ is an element of A. In other words, whether a given Turing machine encoding $\langle M \rangle$ is an element of A depends only on the language $L(M)$ rather than some other aspect of M.

Recall Rice's theorem: Every non-trivial property A of recognizable languages is undecidable. In class we proved this theorem under the assumption that $\emptyset \notin A$. (Clarification: What I mean is that $\langle R \rangle \notin A$, where R is a Turing machine such that $L(R) = \emptyset$.) I asserted that the case where $\emptyset \in A$ (meaning, $\langle R \rangle \in A$) can be proven similarly.

A. Consider problems 5.9, 5.10, 5.11, 5.12, 5.13, 5.32a, 5.32b. In each of these problems, you are asked to prove that a language is undecidable. Which of these undecidability results is an immediate consequence of Rice's theorem? (I am not asking you to prove the other ones.)

B. Where does the proof given in class break, if A is not a property of recognizable languages, but merely some set of Turing machine encodings $\langle M \rangle$?

C. Prove Rice's theorem for the case where $\emptyset \in A$.