You are allowed one two-sided sheet of notes, per my earlier instructions. You are not allowed to use books, calculators, computers, etc.

You should always explain and justify your answers. Correct answers with no explanation may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit.

It is understood that efficient, concise solutions are usually favored over inefficient or verbose solutions, and hence may earn more points.

If you feel that a problem is ambiguously worded, then ask for clarification. If the problem is still unclear, then explain your interpretation in your solution. Never interpret a problem in a way that renders it trivial.

You have 150 minutes. Good luck. :)

A. We have defined a language B to be $PSPACE$-hard if, for all $A \in PSPACE$, there exists a polynomial-time Turing machine F such that $w \in A \Leftrightarrow F(w) \in B$. That's strange — to define a space-related concept in terms of time. Why didn't we require F to be polynomial-space?

B. The Cook-Levin theorem (which says that SAT is NP-complete) is proved by writing a Boolean formula that describes a tableau. Why did we not mimic that proof, in our proof of Savitch's theorem (which says that $NSPACE(s(n)) \subseteq SPACE(s(n)^2)$)?

C. Let A be the set of all strings $\langle M, N \rangle$ such that M and N are Turing machines (over the same input alphabet Σ) and $L(M) = L(N)$. Is A decidable? Prove your answer.

D. Draw a Venn diagram showing how the following 11 sets of languages are (or are not) contained within one another.

CONTEXTFREE (the context-free languages)

CORECOGNIZABLE (the languages A such that $\overline{A}$ is recognizable)

DECIDABLE (the decidable languages)

EXPSPACE

EXPTIME

NP

NPSPACE

P

PSPACE

RECOGNIZABLE (the recognizable languages)

REGULAR (the regular languages)

E. In the USA, (simplified) postal addresses are formatted as in the two examples below. The first line is the addressee: an arbitrary string containing no carriage returns (ASCNCR). The second line is either a street address or a P.O. box. A street address is one or more digits followed by an ASCNCR. A P.O. box is "PO Box" followed by one or more digits. The third line consists of a city name (an ASCNCR with no commas), followed by a comma, followed by a two-letter state/territory code, followed by a ZIP code. The ZIP code is either five digits or five digits followed by a dash and four digits. Write a regular expression for postal addresses as just described. To clarify your work, add comments, define sub-expressions, etc.

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F. Let $A = \{a^n b^n c^n : n \geq 0\} \subseteq \{a, b, c\}^*$. Is A regular? Is A context-free? Prove your answers (without simply citing them, if that is even possible).