

Consider our old friend  $A = \{0^m 1^m : m \geq 0\}$ . I'm going to describe three deciders for  $A$ . For each one, please describe its time and space complexity. Here's the first decider:

1. Check that the input is a string in  $L(0^*1^*)$ .
2. Repeatedly scan the tape, blanking one 0 and one 1 per pass.
3. If the tape ends up empty, then accept. If an error occurs during a pass (because there is a 0 without a matching 1 or *vice-versa*), then reject.

Here's the second decider:

1. Check that the input is a string in  $L(0^*1^*)$ .
2. Repeat until there are no more 0s or no more 1s:
  - (a) Scan the tape. If the total number of 0s and 1s is odd, then reject.
  - (b) Scan the tape, blanking every second 0 and every second 1. (That is, blank the first, third, fifth, etc. 0 and the first, third, fifth, etc. 1.)
3. If the tape ends up empty, then accept. If the tape is not empty, then reject.

Here's the third decider, which uses a two-tape Turing machine:

1. Check that the input is a string in  $L(0^*1^*)$ .
2. Copy the 0s from the first tape to the second tape.
3. Scan the first and second tapes simultaneously, checking that each 1 on the first tape has a corresponding 0 on the second tape. If so, then accept; if not, reject.