You are allowed to use your class notes, your textbook, and the materials on our course web site, including the homework assignments. You are not allowed to share these materials with another student. You are not allowed to consult other books, Web sites, etc. Of course you are not allowed to confer with anyone.

Except where otherwise noted, you must justify your answers. Correct answers with no justification may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit.

You may cite material (definitions, theorems, algorithms, etc.) discussed in class, assigned homework, or the assigned textbook sections. If you wish to use other material, then you must develop it first.

It is understood that efficient, concise solutions are usually favored over inefficient or verbose solutions, and hence may earn more points.

If you feel that a problem is ambiguously worded, then ask me for clarification over e-mail. If the problem is still unclear, then explain your interpretation in your solution. Never interpret a problem in a way that renders it trivial.

A "Turing machine" is understood to be deterministic, unless non-determinism is explicitly specified.

From the time when you first access the exam on Moodle, you have 180 minutes to submit your solutions via Moodle.

Some students will write their solutions on paper and upload photographs or scans. (Use plenty of light!) Some students will type their solutions and upload a PDF or plain text file. Microsoft Word documents are not acceptable. Some students will write on tablets and upload files from there; they must be legible. Let me know if you have other plans.

Good luck. :)

For problems A and B, we say that a string s is a *prefix* of a string t if there exists a string r such that $t = sr$. Over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, let

$$A = \{t\mathtt{c}s : t \in \{\mathtt{a}, \mathtt{b}\}^*, s \in \{\mathtt{a}, \mathtt{b}\}^*, \text{ and } s \text{ is a prefix of } t\}\,.$$

A. Using the pumping lemma for context-free languages, prove that the language A defined above is not context-free.

For problem B, recall what we mean by Moderate Detail in describing Turing machines: You do not draw the Turing machine, but you do describe how the tape head moves across the tape, how the tape head marks or otherwise modifies the tape, and how the Turing machine decides to accept or reject. Usually there is a sequence of left-to-right or right-to-left scans involved. A fellow student, who sees your description at this detail level, should in principle be able to produce a complete drawing of the Turing machine.

B. Design a Turing machine at Moderate Detail, that decides the language A defined above.

For problem C, define

$$B = \{\langle M \rangle : M \text{ is a Turing machine that halts on input } \varepsilon\}.$$

C.A. Is the language B defined above decidable? Prove your answer.

C.B. Is B recognizable? Prove your answer.

C.C. Is the complement B^c recognizable? Prove your answer.