Notes, book, etc. are not allowed.

Except where otherwise noted, you must justify your answers. Correct answers with no justification may receive little credit. Incorrect or incomplete answers that display insight often receive partial credit.

You may cite material (definitions, theorems, algorithms, etc.) discussed in class, assigned homework, or the assigned textbook sections. If you wish to use other material, then you must develop it first.

It is understood that efficient, concise solutions are usually favored over inefficient or verbose solutions, and hence may earn more points.

If you feel that a problem is ambiguously worded, then ask for clarification. If the problem is still unclear, then explain your interpretation in your solution. Never interpret a problem in a way that renders it trivial.

You have 150 minutes. Good luck. :)

A. In each part of this problem, your job is to give a specific example of a language A that meets certain criteria. If there is no such A, then say so. If it is not known whether there is such an A, then give your best answer. Justification is optional.

A.A. Give a regular A that is not finite.

A.B. Give a context-free A that is not regular.

A.C. Give a decidable A that is not context-free.

A.D. Give a recognizable A that is not decidable.

A.E. Give an $A \in \text{NP}$ such that $A \notin \text{P}$.

A.F. Give an $A \in \text{PSPACE}$ such that $A \notin \text{NPSPACE}$.

B. We know that P $\subseteq$ PSPACE. Do we know whether P = PSPACE? Discuss.

C. One way to show that MAX-CUT $\in$ NP is to exhibit a polynomial-time verifier V for it. What would V do? (Be precise. This problem is intended to test your understanding of both verifiers and MAX-CUT.)

D. Consider the Turing machine D that, on input $\langle M, N \rangle$, where M and N are DFAs:

1. Computes $p = |Q_M| \cdot |Q_N|$ (the product of the numbers of states in M and N).
2. For each string x of length not exceeding p, runs M and N on input x for p steps. If ever an x is accepted by one DFA and not the other, then rejects.
3. Accepts.

D.A. What is $L(D)$? Explain.

D.B. Argue either that the space complexity of D is polynomial or that it is not polynomial.

E. Let $A = \{\langle M \rangle : M \text{ is a TM of time complexity } \mathcal{O}(n^2)\}$. Prove that A is not recognizable by reducing $\text{HALT}_{\text{TM}}^c$ (the complement of HALT_{TM}) to A.