I'm trying to set office hours that are useful to my students. By 11:59 PM tonight, please send me e-mail listing *by letter* all of the times, at which you *can* meet, because you do not have a class or work conflict. For example, your e-mail might say "A, C, E, I, J, K, L".

A. Mon 11:10-12:20 (3A)		**B**. Mon 1:50-3:00 (5A)
C. Wed 11:10-12:20 (3A)		**D**. Wed 1:50-3:00 (5A)
E. Fri 12:00-1:00 (3A)	**F**. Fri 1:10-2:10 (4A)	**G**. Fri 2:20-3:20 (5A)
H. Tue 11:00-12:00	**I**. Tue 1:00-2:00	**J**. Tue 2:00-3:00
	K. Thu 1:00-2:00	**L**. Thu 2:00-3:00

Please attempt the following four problems immediately, so that you are prepared for Day 04 (Wednesday). If you have questions, then talk to me in office hours. I'm here to help! Submit all of your solutions on paper at the start of class on Day 05 (Friday).

First. (This is a special case of A.5.1 from the complex.pdf tutorial.) Recall that the Hermitian inner product is the function that takes two input vectors $|\chi\rangle$, $|\omega\rangle$ and produces as output the scalar $\langle\chi|\,|\omega\rangle = |\chi\rangle^*\,|\omega\rangle$. This function is linear in the second argument, meaning

$$|\chi\rangle^*\left(|\omega\rangle + |\psi\rangle\right) = |\chi\rangle^*\,|\omega\rangle + |\chi\rangle^*\,|\psi\rangle \text{ and } |\chi\rangle^*\left(\sigma\,|\omega\rangle\right) = \sigma\,|\chi\rangle^*\,|\omega\rangle\,.$$

It is also conjugate-symmetric, meaning

$$|\chi\rangle^*\,|\omega\rangle = \overline{|\omega\rangle^*\,|\chi\rangle}.$$

Using these two facts, explain why the Hermitian inner product is conjugate-linear in the first argument, meaning

$$\left(|\chi\rangle^* + |\omega\rangle^*\right)|\psi\rangle = |\chi\rangle^*\,|\psi\rangle + |\omega\rangle^*\,|\psi\rangle \text{ and } \left(\sigma\,|\chi\rangle\right)^*|\omega\rangle = \overline{\sigma}\,|\chi\rangle^*\,|\omega\rangle\,.$$

[I corrected a typo in this problem after class on Day 04. While correcting the typo, I changed to a notation that makes the problem easier. You can now do most of the problem by citing basic facts about matrix multiplication. In any event, you need to come out of this experience knowing that (except in special cases) the inner product of $\sigma\,|\chi\rangle$ with $|\omega\rangle$ does not equal σ times the inner product of $|\chi\rangle$ with $|\omega\rangle$.]

(The next two problems a special case of A.5.2 from the complex.pdf tutorial.) Let $|\chi\rangle\,, |\omega\rangle \in \mathbb{C}^2$. The Cauchy-Schwarz inequality says that

$$|\langle\chi|\omega\rangle|^2 \leq \langle\chi|\chi\rangle \cdot \langle\omega|\omega\rangle,$$

and the triangle inequality says that

$$\||\chi\rangle + |\omega\rangle\| \leq \||\chi\rangle\| + \||\omega\rangle\|\,.$$

Second. Prove the Cauchy-Schwarz inequality. (Hint: If $|\chi\rangle$ is the zero vector, then check that the inequality holds. If $|\chi\rangle$ is not zero, then let $|\psi\rangle = |\omega\rangle - (\langle\chi|\omega\rangle / \langle\chi|\chi\rangle)\,|\chi\rangle$, and use the fact that $\langle\psi|\psi\rangle \geq 0$.)

Third. Use the Cauchy-Schwarz inequality to prove the triangle inequality. (Hint: First prove that $\||\chi\rangle + |\omega\rangle\|^2 \leq (\||\chi\rangle\| + \||\omega\rangle\|)^2$. Then you're almost done.)

Fourth. Read the complex.py tutorial. Submit Exercises A and B on paper.