

A. Is the following identity true? Prove or disprove that, for all $|\chi\rangle, |\phi\rangle, |\omega\rangle, |\psi\rangle \in \mathbb{C}^2$,

$$|\chi\rangle \otimes |\phi\rangle + |\omega\rangle \otimes |\psi\rangle = (|\chi\rangle + |\omega\rangle) \otimes (|\phi\rangle + |\psi\rangle).$$

B. Prove that, if $\| |\chi\rangle \| = 1$ and $\| |\phi\rangle \| = 1$, then $\| |\chi\rangle \otimes |\phi\rangle \| = 1$. (So the tensor product of one-qbit states is a legitimate two-qbit state.)

C. To the Resources section of the course web site, I have added a new part, which contains a few Python files. These files are the start of our Python library for simulating quantum computations. Over the coming weeks we will add to them, and we will add other files.

Download and read these files. You are not expected to understand the implementation details of every function, but you are expected to know what tools are available, so that later you can say, “Oh yeah, I think we have some code that does that.”

D. In lecture I claimed that almost all two-qbit states $|\psi\rangle$ violate the unentanglement equation $\psi_{00}\psi_{11} = \psi_{01}\psi_{10}$. In principle at least, you can test my claim in Python by generating a large number of states and, for each one, testing whether it satisfies the unentanglement equation.

Unfortunately, it’s useless to test whether the equation holds exactly, because the computer’s underlying floating-point calculations are inexact. So you need to consider *by how much* the equation is violated. For example, you could inspect the deciles (0th, 10th, 20th, etc. percentiles) of the mismatch in the equation. If my claim is true, then you will see not just tiny mismatches explainable by numerical imprecision, but also big mismatches.

Using our quantum computing library, perform an experiment like the one described above. For example, you will want to call `qu.uniform(2)` repeatedly. Print on paper your experimental code, the results, and a brief discussion of whether the results support or refute my claim.