

A. Find the periods of all elements in $(\mathbb{Z}/11\mathbb{Z})^*$.

B. In $(\mathbb{Z}/16\mathbb{Z})^*$, what is $\log_5 13$, and what is $\log_5 11$?

In this part of the course, we continue to use $\alpha, \beta, \gamma, \dots$ to denote n -bit strings. We also use a, b, c, \dots to denote the integers in $\{0, 1, \dots, 2^n - 1\}$ that correspond to those strings. For example, if $\beta = 1101$, then

$$b = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 8 + 4 + 0 + 1 = 13.$$

C. Let T be the $2^n \times 2^n$ matrix defined by

$$T_{\alpha, \beta} = \frac{1}{2^{n/2}} e^{2\pi i \alpha \beta / 2^n}.$$

Prove that T is unitary.