For most students, this exam is due on paper at the start of class on Monday. For students who have made special arrangements with me, the exam is due on paper in my mail slot (in the Math/Stats Department suite on the second floor of CMC) at noon on Tuesday.

The exam must be completed within a single 180-minute (three-hour) block of time. Your block begins when you open (or peek inside) this packet.

The exam is open-note: You may use your class notes, your old homework and Python work, and the course web/Moodle site. You may not share any of these materials with other students.

The exam is closed-book: You may not use the textbook, any of the reserve books, any other books, papers, Internet sites, etc. You may not discuss the exam with anyone but me until Tuesday at noon.

I will try to check my e-mail frequently during the exam period. Feel free to ask clarifying questions. If you cannot obtain clarification on a problem, then explain your interpretation of the problem in your solution. Never interpret a problem in a way that renders it trivial. Check your e-mail occasionally, in case I send out a correction.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. Always show enough work and justification so that a typical classmate could understand your solutions. You may cite material (definitions, theorems, examples, algorithms, etc.) from class, homework, etc. You do not have to redevelop or reprove that material. If you wish to use material that we haven't studied, then you have to develop it. Correct answers without supporting work rarely earn full credit.

If you cannot solve a problem, then write a brief summary of what you know that is relevant, and the approaches that you've tried. Partial credit is often awarded.

Good luck. :)

There are five problems below. It might help you to know that I regard problems A, B, and D as short and problems C and E as medium-length. In other words, on no problem do I expect a gigantic amount of work.

A. Define a three-qbit quantum gate S to act like this on the classical states:

$$S|\alpha_2\alpha_1\alpha_0\rangle = |\alpha_0\alpha_2\alpha_1\rangle.$$

Write the matrix for S.

B. I start with a certain classical function f. I convert it into a quantum gate F in the usual way. Here is what the resulting six-qbit F does to classical states:

$$F \cdot |\alpha_5\alpha_4\alpha_3\alpha_2\alpha_1\alpha_0\rangle = |\alpha_5\rangle \otimes |\alpha_4\rangle \otimes |\alpha_3\rangle \otimes |\alpha_2\rangle \otimes |\alpha_1 \oplus (\alpha_5 \odot \alpha_3)\rangle \otimes |\alpha_0 \oplus (\alpha_4 \odot \alpha_2)\rangle.$$

What is f? (A complete answer makes clear (A) what the domain is, (B) what the codomain is, and (C) what the formula is, for how elements of the domain are sent to the codomain. I am *not* asking you (D) what the meaning of f is in English.)

C. The Bernstein-Vazirani algorithm starts with input $|0\cdots 0\rangle \otimes |1\rangle$. What if it instead started with $|0\cdots 0\rangle \otimes |0\rangle$? Would it solve the same problem, or solve some other problem, or do nothing of value?

D. One day, while running Simon's algorithm, I build this matrix of γ-values in reduced row-echelon form:

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

What is δ?

E. Suppose that you had an algorithm for quickly solving the period-finding problem, that worked only when the period p was odd. The algorithm would correctly output p when p was odd, but it would incorrectly output 1 whenever p was even. How could you use this algorithm, to make a new algorithm that quickly solves the period-finding problem for all p? In your answer, discuss the running time of the new algorithm relative to the old one.