

There are eight problems. When you are asked to prove something, *prove* means “give an explanation that would convince a skeptical classmate”. A picture often helps.

1. Section 12.2 #19.
2. Section 12.2 #23.
3. Section 12.2 #30.
4. Section 12.2 #45. [For some students, this problem foreshadows the first week of Math 232. But you don't need to have studied linear algebra, to solve it.]
5. Let \vec{p} and \vec{q} be any two points in \mathbb{R}^3 . Let $\vec{m} = \frac{1}{2}(\vec{p} + \vec{q})$. Using basic properties of vector addition and scalar multiplication, prove that \vec{m} is the midpoint of the line segment between \vec{p} and \vec{q} . [Hint: Compare $\vec{p} - \vec{m}$ to $\vec{q} - \vec{m}$. And draw a picture!]
6. Section 12.2 #51.
7. Section 12.2 #7.
8. In any given year, a company's *revenue* is its income — that is, the amount of money that it pulls in. (The profit is the revenue minus the cost.) Let's denote the revenue as r . Now consider an industry made up of two companies with revenues r_1 and r_2 respectively. The overall state of the system can be represented as a vector $\vec{r} = \langle r_1, r_2 \rangle$ pointing to the point (r_1, r_2) in the first quadrant of \mathbb{R}^2 . Because revenue changes from year to year, this point \vec{r} moves around, as time passes. It traces out a curve (or at least a sequence of dots) in the first quadrant.
 - (a) If the two companies are competing for the same fixed pool of consumers, then how might the curve look? Draw a picture and briefly explain.
 - (b) On the other hand, if the two companies are in cooperation — for example, one supplies the other with materials — then how might the curve look?
 - (c) Re-do those first two questions for the case where there are three companies.

Epilogue: Many industries contain more than three companies, so problems like this require many dimensions. In general, higher dimensions are not a fantasy created to entertain mathematicians. Lots of people use them every day.

If you feel that you need more practice with this foundational vector material, then do more problems from the book, or do problems from other calculus textbooks in the Math Skills Center, or come talk to me. :)