1. Section 12.4 #2.
2. Section 12.4 #3.
3. Section 12.4 #13.
4. Section 12.4 #16.
5. Section 12.4 #26.
6. (By the way, this problem is similar to #44.) Suppose that $\vec{v}$ and $\vec{w}$ are non-zero and perpendicular. Find a vector $\vec{x}$, in terms of $\vec{v}$ and $\vec{w}$, such that $\vec{v} \times \vec{x} = \vec{w}$. Also, why is it important to this problem that $\vec{v}$ and $\vec{w}$ be perpendicular? Also, why is it important that $\vec{v}$ and $\vec{w}$ be non-zero?
7. In robotics, astronomy, computer graphics, and other fields, we need to be able to compute how objects revolve around each other. To that end, let $\vec{u}$ be a unit vector in $\mathbb{R}^3$, let $\vec{v}$ be any vector in $\mathbb{R}^3$, let α be any angle, and let

$$\vec{w} = (\vec{u} \cdot \vec{v})(1 - \cos\alpha)\vec{u} + (\cos\alpha)\vec{v} + (\sin\alpha)\vec{u} \times \vec{v}.$$

Imagine that $\vec{u}$ is an axis of rotation (like an axle on a car). Place $\vec{v}$ tail-to-tail with $\vec{u}$. Rotate $\vec{v}$ about $\vec{u}$ in a right-handed manner through the angle α. I claim that the resulting vector is the $\vec{w}$ defined above. Your job is to partially verify this claim, by doing this four-part problem:

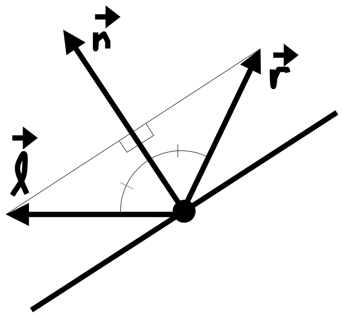
   (a) Check algebraically that $\vec{w} \cdot \vec{u} = \vec{v} \cdot \vec{u}$.

   (b) Check algebraically that $|\vec{w}| = |\vec{v}|$.

   (c) Draw a picture of what's going on. Your picture should include annotations explaining the geometric meanings of the first two parts of this problem.

   (d) Is there anything else that should be checked, to verify the claim? (I'm not asking you to check it. I'm just asking you to describe in English what needs to be checked, if anything.)

8. The picture below visualizes the operation of reflecting a vector $\vec{\ell}$ (of any length) across a unit vector $\vec{n}$, which is normal to a surface, to obtain a reflected vector $\vec{r}$. This operation is common in computer graphics; for example, it's a key step in simulating how rays of light bounce off a mirror. Prove that $\vec{r} = 2(\vec{n} \cdot \vec{\ell})\vec{n} - \vec{\ell}$. (Hint: Use a vector projection.)



Homework constitutes the preceding eight questions. What follows is an optional ninth question. You are not required to hand it in. It is not worth extra credit. I provide it to you, just in case you want some more practice.

Planets, moons, comets, etc. tend to move in elliptical orbits. However, in this problem we are dealing with such a small segment of orbit, that we can reasonably approximate it as a straight line. Also, let's assume that our universe is two-dimensional (although the problem is not really any harder in three dimensions).

An astronomer working late at night notices a previously unknown asteroid on her computer. The asteroid is at position $\vec{p}$ and moving with velocity $\vec{v}$. (The unit of distance is 10^6 m, measured relative to the Sun, and the unit of time is the hour. Time $t = 0$ is when the astronomer discovers the asteroid.) At that moment, Earth is at position $\vec{q}$ and moving with velocity $\vec{w}$.

a) Write expressions, in terms of $\vec{p}$, $\vec{v}$, $\vec{q}$, $\vec{w}$, and t, for these three quantities: the position of the asteroid at time t, the position of the Earth at time t, and the distance between the two bodies at time t (regarding them as point particles).

b) Still working in terms of $\vec{p}$, $\vec{v}$, $\vec{q}$, $\vec{w}$, and t, find the time t at which the two bodies are closest, and how close they are at that time. (Hint: Minimize the distance squared. Also, are you able to do it without calculus?)

c) Now let's plug in some concrete numbers:

$$\begin{aligned}
\vec{p} &= (213268.00, 208956.00), \\
\vec{v} &= \langle 56.66, 113.32 \rangle, \\
\vec{q} &= (212132.00, 212132.00), \\
\vec{w} &= \langle 75.66, -75.66 \rangle.
\end{aligned}$$

Earth's radius is 6.38 and the asteroid's radius is 0.1. Will the asteroid hit Earth?

d) Using the same values for $\vec{p}$, $\vec{v}$, $\vec{q}$, $\vec{w}$ as in the previous part, where will the asteroid be, 12 hours after its discovery by the astronomer? Where will Earth be, 24 hours after the discovery?