Homework consists of these nine problems from the book:

Section 15.4 #3, 18, 29, 30.

Section 15.6 #2, 3, 9, 15, 37.

Two three-part bonus problems are below. They are not required, they will not be graded, and they are not worth extra credit, but they might improve your calculus skills.

1. Richard Feynman (1918–1988) was one of the most successful (and colorful, and controversial) physicists of the 20th century. In his memoirs, he gleefully revealed his favorite mathematical trick: "differentiation under the integral sign". Basically, it amounts to pushing a differentiation with respect to one variable into an integral with respect to a different variable:

$$\frac{d}{dx}\int_c^d f(x,y)\,dy = \int_c^d \frac{\partial}{\partial x} f(x,y)\,dy.$$

It works for functions f that are "sufficiently nice". Now here's a three-part problem.

A. For any $a > 0$, let

$$I(a) = \int_0^\infty \frac{e^{-x} - e^{-ax}}{x}\,dx.$$

Assume that every function in this problem and its solution is sufficiently nice. Show that $I(a) = \log a$ (meaning the base-e logarithm).

B. You should wonder what it means for f to be "sufficiently nice". Well, here's part of it. Suppose that there exists a function $g(u,y)$ such that

$$f(x,y) = \int_h^x \int_a^t g(u,y)\,du\,dt.$$

Show that f is a second partial anti-derivative of g with respect to x. In other words, show that

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x} f(x,y) = g(x,y).$$

(Hint: You need the fundamental theorem of calculus.)

C. Prove that differentiation under the integral sign works for sufficiently nice f. (Hint: You need the assumption on f from part B. You also need Fubini's theorem for triple integrals. In applying it, you will make an additional assumption on f or g. Then you will have a more thorough understanding of what it means for f to be sufficiently nice.)

2. Recall that we used the Mathematica notebook `integration.nb` on Day 15 of the course. The notebook ends with the example of

$$\iint_R \frac{x-y}{x^3+y^3}\,dA,$$

where $R = [0, 1] \times [0, 1]$.

A. What does Mathematica give for the value of the integral, computed in the $dx\, dy$ order and in the $dy\, dx$ order?

B. By mimicking how I used the `integrate` (not `Integrate`!) function in earlier examples, compute a Riemann sum approximation to the integral. You might want to try a couple of values for `n`. Show the line(s) of code that you used and the result(s).

C. Explain how these results relate to Fubini's theorem.