Section 16.2 #3, 11, 19, 22, 49, 54.

This final, multi-part problem is unusually important, because it presents a crucial example that will refine your understanding of this part of Math 211. The problem is not computationally difficult, but it is conceptually difficult. You need to be careful about things that are usually simple and boring, such as "Where is this function defined?" and "$+\ C$". Anyway, let D be the set of points in the plane other than the origin:

$$D = \{(x, y) : x \neq 0 \text{ or } y \neq 0\}.$$

Also let

$$\vec{F}(x, y) = \langle P(x, y), Q(x, y)\rangle = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle.$$

Notice that $\vec{F}$ is defined on all of D. It's okay that $\vec{F}$ is not defined at the origin; we care only about $\vec{F}$ on D.

A. Show that $\frac{\partial}{\partial y} P = \frac{\partial}{\partial x} Q$ everywhere on D. (So our usual test does not preclude the existence of a potential function.)

B. Find all potential functions for $\vec{F}$ on the part of D where $x > 0$. Your answer should involve an unknown constant C_1. (Hint: Consider something like $\arctan(y/x)$ or $\arctan(-x/y)$.)

C. Find all potential functions for $\vec{F}$ on the part of D where $y > 0$. Your answer should involve a C_2.

D. Find all potential functions for $\vec{F}$ on the part of D where $x < 0$, using a C_3.

E. Find all potential functions for $\vec{F}$ on the part of D where $y < 0$, using a C_4.

F. Explain why there is no potential function for $\vec{F}$ on all of D. (Hint: If such a function existed, then what would its values be at the four points $(x, y) = (\pm 1, \pm 1)$?)