You have 60 minutes.

No notes, books, calculators, computers, etc. are allowed.

Except where otherwise noted, show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Perform as much algebraic simplification as you can. Do simple arithmetic, but don't bother to do complicated arithmetic. Mark your final answer clearly.

Remember that I am grading your paper, not you. Make sure your paper says exactly what you want it to say.

Good luck. :)

A. Consider the function $f(x, y, z) = x^2y - 3xz + z^4$ and the surface defined by $f(x, y, z) = -1$.
A.A. Compute $\partial f/\partial x$, $\partial f/\partial y$, and $\partial f/\partial z$.

A.B. Give an equation for the tangent plane to the surface at $(2, 1, 1)$.

A.C. Give an equation for the line perpendicular to the surface at that point.

B. You are hiking along a hill defined by $z = f(x, y) = e^{-x^2-y^2}$, where the x-axis points east, the y-axis points north, and distances are measured in km. After resting at $(1, 0)$, you start walking northwest at 1 km per hour. How quickly is your elevation changing then?

C. In physical chemistry, the *Redlich-Kwong equation of state* says that

$$p = \frac{RT}{V_m - b} - \frac{a}{\sqrt{T} V_m (V_m + b)},$$

where p, T, and V_m are the pressure, temperature, and molar volume of a gas, respectively, and R, a, and b are constants. Suppose that, at a certain moment in your chemistry experiment, p has value 21 and is increasing at 0.3 units per second, T has value 4 and is increasing at 0.1 units per second, and V_m has value 1. How quickly is V_m changing at that moment?

D. A rocket is launched at time 0 with 100,000 liters of fuel. Its engines produce a steady (meaning, unchanging over time) acceleration of a m/s^2, or $(a - 9.8)$ m/s^2 after gravity is taken into account. By time t, the rocket has reached an altitude of $(a - 9.8)t^2/2$ meters (ignoring air resistance), and it has used a^2t liters of fuel. Using an optimization technique *from this course*, find the value of a that maximizes the altitude that the rocket reaches when its fuel runs out.

E. In this problem, $f(x,y)$ is a function of two variables. Each subproblem has four possible answers: TRUE, FALSE, TRUISH, and FALSISH. If the correct answer is TRUE, then TRUE earns 3 points, TRUISH earns 2 points, FALSISH earns 1 point, and FALSE earns 0 points. If the correct answer is FALSE, then these point values are of course reversed. Do not write just T or F; write the whole answer clearly. No explanation is needed (or graded)!

E.A. If f_{xy} and f_{yx} are continuous on a disk about (a,b), then they must be equal at (a,b).

E.B. If f has a local min or max at (a,b), then it must be true that $\nabla f(a,b) = \vec{0}$.

E.C. If $\nabla f(a,b)$ exists, then f must be differentiable at (a,b).

E.D. If $\lim_{(x,y)\to(a,b)} f(x,y)$ and $f(a,b)$ exist and are equal, then f must be continuous at (a,b).

E.E. If $\nabla f(a,b) = \vec{0}$ and $f_{xx}(a,b) > 0$, then f must have a local min at (a,b).

E.F. The global (absolute) max of f on the disk $\{x^2 + y^2 \leq 1\}$ must occur at one of f's critical points.