You have 60 minutes.

No notes, books, calculators, computers, etc. are allowed.

When you are asked to "prove" something, you are being asked to give a convincing explanation of why that fact is true. Imagine that your audience is a skeptical classmate.

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Pictures often help both you and your reader!

Perform as much algebraic simplification as you can. Simple correct answers are generally preferred over complicated correct answers. Do simple arithmetic, but don't bother to do complicated arithmetic. Mark your final answer clearly.

Good luck. :)

A. Let $\vec{v} = \langle 2, 1, -1 \rangle$ and $\vec{w} = \langle -4, 1, 2 \rangle$. Calculate the following quantities.

A.A. The vector from the point $\vec{v}$ to the point $\vec{w}$:

A.B. $\text{proj}_{\vec{w}}\vec{v}$:

A.C. The vector of length 3 in the same direction as $\vec{v}$:

B. In general, if $\vec{v} \times \vec{w} = \vec{u} \times \vec{w}$ and $\vec{w} \neq \vec{0}$, then must it be true that $\vec{v} = \vec{u}$? Explain.

C. Consider the curves $\vec{x}(t) = \left(1 + t^2, \dfrac{\sin t}{t}\right)$ and $\vec{y}(t) = \left(t^3, \dfrac{2t}{\sqrt{t^2+1}}\right)$ in $\mathbb{R}^2$.

C.A. $\lim_{t\to 0} \vec{x}(t) =$

C.B. Is $\vec{x}$ continuous at $t = 0$?

C.C. $\vec{x}\,'(t) =$

C.D. $\int \vec{y}(t)\, dt =$

D. This problem is about the curve $y = x^2$ in $\mathbb{R}^2$.

D.A. Express the curve in polar coordinates, with r as a function of θ if possible.

D.B. What is the domain of that function r? In other words, what values of θ should be plugged into the function, to produce the curve?

E. Consider the curve $\vec{x}(t) = (1 + \cos t, 2 \sin t)$, for $0 \leq t \leq 2\pi$, in $\mathbb{R}^2$.

E.A. Sketch this curve as accurately as you can.

E.B. Compute the length of the curve. If you cannot finish, then go as far as you can.

F. In $\mathbb{R}^4$ with coordinates x, y, z, w, consider the point $\vec{p} = (p_1, p_2, p_3, p_4)$ and the "hyperplane" given by $ax + by + cz + dw = e$. Find an expression, in terms of a, b, c, d, e, p_1, p_2, p_3, and p_4, for the distance from the point to the hyperplane. (Hint: The argument is similar to what you would do in $\mathbb{R}^2$ or $\mathbb{R}^3$.)