This exam is due on paper at the start of class on Friday. It must be completed within a single 120-minute (two-hour) block of time. Your block begins when you open (or peek inside) this packet.

The exam is open-book and open-note: You may use the official class textbook, your class notes, your old homework, and your Exam A. You may not share any of these materials with other students.

You may e-mail me for clarification on any problem. I will try to check my e-mail frequently. If you do not receive clarification, then explain your interpretation of the problem as part of your solution. If you have found an interpretation that renders the problem vacuous or trivial, then that is the wrong interpretation.

You may not use any other books, Internet sites, software, calculator, computer, phone, etc. You may not discuss the exam with anyone but me.

Your solutions should be thorough, self-explanatory, neat, concise, and polished. Always show enough work and justification so that a typical classmate could understand your solutions. If you cannot solve a problem, then write a brief summary of what you know that is relevant, and the approaches that you've tried. Partial credit is often awarded.

Good luck. :)

There are four problems. It might help you to know that I regard them as short, medium, medium, and long, respectively.

A. Consider the function

$$f(x, y) = \frac{x^2 + xy^2}{x^4 + y^2}.$$

Does its limit at the origin exist? Explain.

B. Recall that, relative to a large mass $M > 0$ at the origin in $\mathbb{R}^3$, the gravitational potential energy of a small mass $m > 0$ at position $\vec{x}$ is

$$V = -\frac{GMm}{|\vec{x}\,|},$$

where G is a positive constant. What is the rate of change of V, when the small mass at $\vec{x}$ is moving toward the origin at speed 1?

C. Suppose that y is a function of $\vec{x}$, which is a function of t. (Here, $\vec{x}$ is in $\mathbb{R}^n$ for some unspecified dimension n.)

C.A. Compute $\frac{d}{dt}y$. Concise, simple answers are preferred over long, complicated ones.

C.B. Compute the second derivative $\frac{d}{dt}\frac{d}{dt}y$. Concise, simple answers are preferred.

C.C. Check your answer to part B on the two-dimensional example $y = x_1 x_2$, $\vec{x} = (t^2, \sin t)$.

D. You are planning to open a store in a certain city. You want the store to be within 2 km of the city center. Beyond that requirement, you want to put the store at the location that maximizes your profit. Your profit is your revenue minus your costs. To describe those quantities, let's put coordinates on the city, with x- and y-axes running east and north, respectively, from the city center, with units of km. If you place your store at (x, y), then your revenue is predicted to be

$$r = 10 - \frac{1}{2}(x-1)^2 - \frac{1}{2}(y-1)^2$$

(because your customer demographics are best in the northeast). Your cost is predicted to be

$$c = 3 - \frac{3}{8}x^2 + \frac{1}{16}x^3$$

(because your cost is mainly rent, and rents are high near the city center but otherwise generally increase to the east).

D.A. Formulate the profit maximization problem in terms of x and y using techniques from this course. Be clear about the function being optimized and any relevant region or constraint.

D.B. For the optimization problem from part A, which subproblems do you need to solve? Which equations need solving, which points need checking, etc.? Be thorough.

D.C. Solve the problem following your plan from part B. (Some of it is difficult without a calculator. Go as far as you can. Getting the final answer is worth very few points.)