You have 70 minutes.

No notes, books, calculators, computers, etc. are allowed.

**Sometimes you are asked to "write a computable integral". This phrase means that you should write an integral but not compute its value. Your integral should be set up so that, if you were to compute it, that computation would be as easy as possible. The "integral" can be a sum of one or more integrals, if that is necessary for the problem.**

Show all of your work, in as organized a manner as possible. Incorrect answers with solid work often earn partial credit. Correct answers without explanatory work rarely earn full credit.

Pictures often help both you and your reader!

Perform as much algebraic simplification as you can. Simple correct answers are generally preferred over complicated correct answers. Do simple arithmetic, but don't bother to do complicated arithmetic. Mark your final answer clearly.

Good luck. :)

**A**. Let $A$ and $B$ be constants such that $0 < A < B$. In $\mathbb{R}^3$, consider the region $E$ trapped between the surfaces $x^2 + y^2 + z^2 = A^2$ and $x^2 + y^2 + z^2 = B^2$.

**A.A**. Write a computable triple integral for $f(x, y, z) = (x^2 + y^2 + z^2)^{-3}$ over $E$.

**A.B**. Compute the integral.

**B**. As a consultant in the energy industry, you're considering damming a river to form a reservoir that can be used for energy storage. You place coordinates so that the base of the proposed dam is at the origin, the $x$-axis runs upstream, and the $z$-axis is vertical. If your dam has height $h$, and the river valley upstream from the dam fills with water, then the water forms a tetrahedron with vertices $(0,0,0)$, $(0,2h,h)$, $(0,-2h,h)$, and $(40h,0,h)$. Two of the planes bounding this tetrahedron are $-x-20y+40z=0$ and $-x+20y+40z=0$. The density of water is 1,000 kg/m$^3$.

Write a computable integral that answers this question: If you release all of that water onto perfectly efficient dynamos at the base of the dam, then how much energy do you recover?

**C**. Let $\sigma > 0$ be a constant. Suppose that two random variables $X$ and $Y$ have joint probability density function

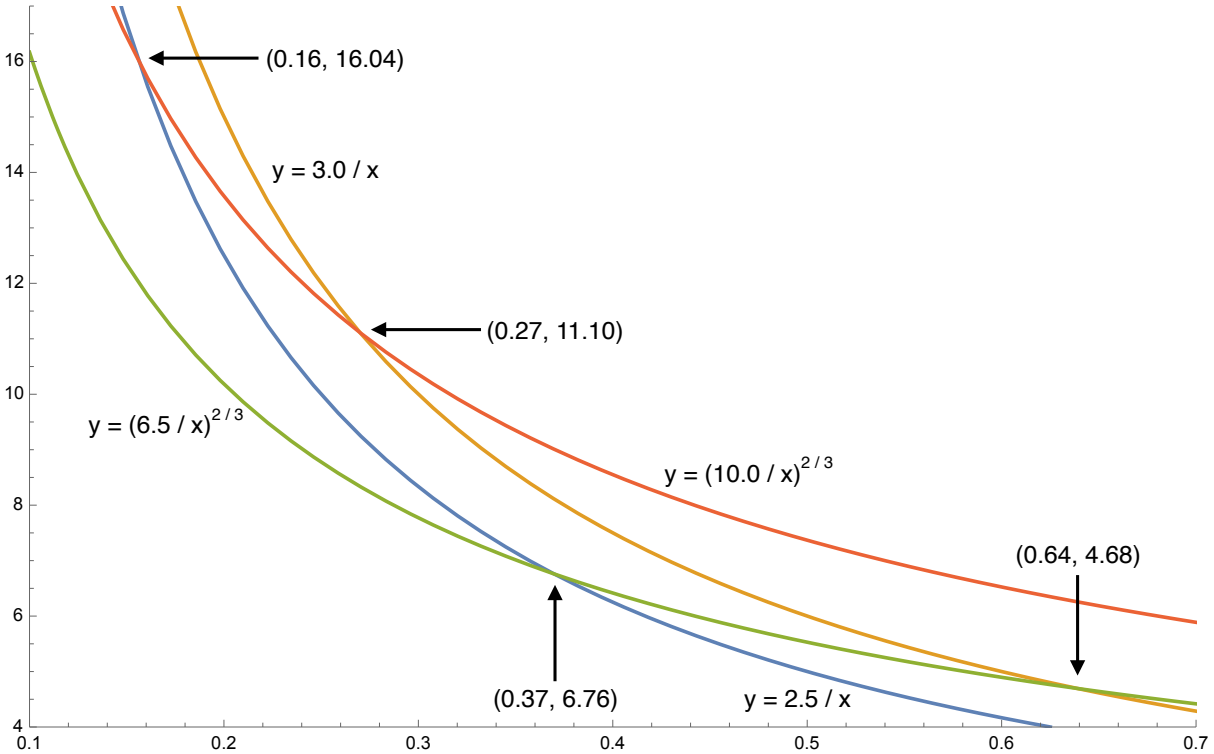
$$f(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)}.$$

The probability that $X \geq 0$, $Y \geq 0$, and $X^2 + Y^2 \geq 1$ is the integral of $f$ over the region of the $x$-$y$-plane described by those three inequalities.

**C.A**. Write a computable integral for this probability.

**C.B**. I am not asking you to compute the integral, but if I did ask you to compute it, then how would you do it, in plain English? Which technique(s) would you use?

**D**. Here is a problem inspired by thermodynamics. The plot below shows two "isothermal" curves and two "adiabatic" curves. They bound a curved, four-sided region, whose four vertices are listed. The area of this region is important (because it measures thermodynamic work). Write a computable double integral for the area. (A Calculus-1-style single integral is unacceptable.)



**E**. The Great Pacific garbage patch (GPGP) is a region of the North Pacific Ocean, where currents have caused plastic debris to accumulate. The GPGP is circular with radius 700 km. The density of plastic at horizontal distance $d$ km from the center and depth $D$ km below the surface has been modeled as $Cd^{-1}D^{-1/2}$ kg/km$^3$, where $C = 5{,}084$.

**E.A**. Write a computable integral for the plastic mass between the surface and a depth of 5 km.

**E.B**. Compute the integral.