A.A. Because the region and the integrand are both rotationally symmetric about the origin, it makes sense to proceed in spherical coordinates. There, the surfaces are $\rho = A$ and $\rho = B$, and the integrand is $f = \rho^{-6}$. So the integral is

$$\int_0^{2\pi} \int_0^{\pi} \int_A^B \left(\rho^2\right)^{-3} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \int_0^{2\pi} \int_0^{\pi} \int_A^B \rho^{-4} \sin\phi \, d\rho \, d\phi \, d\theta.$$

A.B. Fortunately, the integral separates as

$$\int_0^{2\pi} d\theta \cdot \int_0^{\pi} \sin\phi \, d\phi \cdot \int_A^B \rho^{-4} \, d\rho = 2\pi \cdot 2 \cdot \left[-\frac{1}{3}\rho^{-3}\right]_A^B = -\frac{4\pi}{3}\left(B^{-3} - A^{-3}\right) = \frac{4\pi}{3}\left(A^{-3} - B^{-3}\right).$$

B. [I'll omit all pictures in these typed solutions, although I used pictures to figure out my solution, and you should too.] The energy recovered equals the work needed to raise the water from $z = 0$. That work is the integral, over the region occupied by the water, of $\delta g z$, where $\delta = 1000$ and $g = 9.8$. So we obtain the integral

$$\int_0^{40h} \int_{x/40}^{h} \int_{x/20-2z}^{2z-x/20} 9800z \, dy \, dz \, dx.$$

[And here's another solution, in the $dy\, dx\, dz$ order:

$$\int_0^{h} \int_0^{40z} \int_{x/20-2z}^{2z-x/20} 9800z \, dy \, dx \, dz.$$

If you use one of the four other orders, then you might have to break the integral into two integrals, which is generally undesirable.]

C.A. This problem seems simplest in polar coordinates. The bounds amount to $0 \leq \theta \leq \pi/2$ and $r \geq 1$, and the $x^2 + y^2$ in the integrand turns into r^2. So the desired integral is

$$\int_0^{\pi/2} \int_1^{\infty} \frac{1}{2\pi\sigma^2} e^{-r^2/(2\sigma^2)} r \, dr \, d\theta.$$

C.B. Fortunately, the integral separates. The $d\theta$ integral is easy. For the dr integral, I would use anti-differentiation by substitution with $u = -r^2/(2\sigma^2)$. This works well with the other r in the integrand. I would need to take a limit as the upper integration bound went to ∞.

D. We need a 2D integral, and I see no reason to switch to polar coordinates, so I stay in rectangular coordinates. Because of how the boundary curves are arranged, I need to break the integral into three pieces:

$$\int_{0.16}^{0.27} \int_{2.5/x}^{(10.0/x)^{2/3}} 1 \, dy \, dx + \int_{0.27}^{0.37} \int_{2.5/x}^{3.0/x} 1 \, dy \, dx + \int_{0.37}^{0.64} \int_{(6.5/x)^{2/3}}^{3.0/x} 1 \, dy \, dx.$$

[This problem is intended to be the easiest problem on the exam. By the way, this problem is similar to Section 15.9#24. We did not study that section of the textbook. If we had, then we would have learned an alternative approach to computing such an integral.]

E.A. Let's place cylindrical coordinates so that the origin is at the center of the GPGP, on the water's surface, with $d = r$ and $D = -z$. The integral is

$$\int_0^{2\pi} \int_0^{700} \int_{-5}^{0} C r^{-1} (-z)^{-1/2} r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^{700} \int_{-5}^{0} 5084 (-z)^{-1/2} \, dz \, dr \, d\theta.$$

E.B. Fortunately, the integral separates as

$$5084 \int_0^{2\pi} d\theta \cdot \int_0^{700} dr \cdot \int_{-5}^{0} (-z)^{-1/2} \, dz = 5084 \cdot 2\pi \cdot 700 \cdot \left[-2(-z)^{1/2} \right]_{-5}^{0} = 5084 \cdot 700 \cdot 4\pi \cdot \sqrt{5}.$$

[This answer is very close to 10^8 kg, which matches estimates of plastic mass that I found online while researching this problem. I rigged C to make this happen. Similarly, the 700 was based on estimates of the GPGP's area, and the 5 was based on estimates of the abyssal plain's depth. The density function was further inspired by vague observations I read, that plastic density is higher near the center and near the surface, but the specific form of the density function is my own invention and might not be realistic. Also, I excluded the plastic that's accumulating on the sea floor, because I lacked information about it.]

[Bonus learning opportunity: Write a problem about the mass of plastic on the sea floor. What might the integrand be? What kind of integration would appear?]